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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured on a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 6: problem solving, including problems on consecutive integers, angles, uniform motion, and mixtures; and adding and subtracting polynomials. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

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PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

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VOLUME

6

SE 015 861

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

ED 075207

P R O G R A M M E D M A T H C O N T I N U U M

LEVEL ONE

A L G E B R A

VOLUME 6

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME 3

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IN THE STUDY GUIDE:

QUESTION:	SEGMENT:	IS ON PAGE:
1	1	$\frac{1}{1}$
1	2	$\frac{47}{1}$
1	3	$\frac{89}{1}$
1	4	$\frac{130}{1}$
1	5	$\frac{160}{1}$

VOLUME 6

This volume covers the following material as shown in this excerpt from the Syllabus:

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION		
		DOLCIANI	DRESSLER	DODES
1	Plan for solving problems	5-4	5-2 , 5-5	5-5 , 5-6
	Problems on consecutive intergers	5-5	10-1 10-2	5-7
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5	Adding polynomials	6-1	(7-1) 8-1	
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READING ASSIGNMENT

VOLUME 6

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	166	167
2	170	175
3	178	180
4	182	183
5	197	198
	200	201

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERTHING contained in these pages.

EXAMINE every illustrative problem.

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

HOMEWORK ASSIGNMENT

VOLUME NO. 6

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	168	4, 6	06110
2	168	7, 10	06110
3	169	13, 14	06110
4	169	18, 19	06110
5	171 Bot.	2, 4, 5,	06210
6	171 Bot.	7, 8	06210
7	177	3, 7	06230
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9	181	2, 5	06310
10	181	7, 9	06310
11	181	10, 11	06310
12	182	16, 19	06310
13	184	3, 4	06410
14	184	5, 6	06410
15	184	8, 9	06410
16	184	11, 12	06410
17	199 Bot.	4, 9, 10	06520
18	199 Bot.	5, 12, 13, 14	06520
19	202	3, 4, 5	06530
20	202	11, 18, 22, 27	06530

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card
it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

VOLUME 6 SEGMENT 1 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	2 6	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 6	(Volume Number)
	66	and	68	0 1	(Segment Number)

Your READING ASSIGNMENT for this segment is pages 166 - 167 .

SUPPLEMENTARY NOTE:

In your study of arithmetic, you encountered problems like " find the product of $3\frac{1}{2}$ and $5\frac{1}{4}$. " You also came across problems like " a boy worked for $3\frac{1}{2}$ hours and finished mowing $\frac{3}{4}$ of a certain lawn; how much longer will it take the boy to complete the job? "

The second type of problem can be called a story problem. In the first type of problem, you were told exactly what to do; in the second type, you have to decide how to solve the problem before you can do the computation.

Verbal problems in algebra are like the story problems of arithmetic. However, they are sometimes too difficult to be solved by arithmetic. You will find that algebra can be used effectively for the solution of such problems. The task consists of finding an algebraic equation that represents the relationships described in words.

This segment will be an exciting adventure where you will discover a great tool for solving a variety of problems - - the algebraic equation.

You will now be asked a series of questions to draw your attention to the more important points.

$\frac{2}{1}$

Question 1

Recognize the algebraic phrase which is a translation of the English phrase: "Six less than a certain number." Select the letter next to the correct answer.

(A) $-6n$

(B) $n - 6$

(C) $6 - n$

(D) $-(n + 6)$

$\frac{2}{2}$

We do not agree. If we let s represent the number of students in your class, then one more student than this number is represented by

$$(s + 1)$$

Now, there are three such classes.

Please continue.

Please return to page $\frac{25}{2}$ and try this question again.

$\frac{3}{1}$

Please note that ten is increased by a certain number n. That is, we start with 10 and increase it by n.

Please return to page $\frac{29}{2}$ and try this question again.

$\frac{3}{2}$

Re-read the problem and see whether it said that the perimeter of the equilateral triangle is 17 inches. If that were the case, then it would be true that each side would be $\frac{1}{3}$ of 17.

Please return to page $\frac{27}{2}$ and consider this problem again.

$\frac{4}{1}$

The expression

$$12 > 2n$$

means " 12 is more than twice a number."

This is certainly not the same as "twelve more than twice a number."

Please return to page $\frac{20}{2}$ and try this question again.

$\frac{4}{2}$

You did not answer the question. Please listen carefully to the following suggestion. When you represent the unknowns, write out in detail what they stand for. For example:

do not say let x = the bottle; but say

let x = the cost of the bottle

let x + 23 = the cost of the liquid.

Please return to page $\frac{19}{2}$ and try this question again.

$\frac{5}{1}$

Translating an English phrase into an algebraic phrase is like following a recipe, or like obeying a series of commands. First, we replace the word "number" by some letter, say, "n" .

Then, we follow the orders given, one phrase at a time.

Please return to page $\frac{18}{2}$ and try this question again.

$\frac{5}{2}$

The equation you chose is fundamentally correct. It does reflect the conditions of the problem. However, the equation, as it stands, cannot be solved without another substitution for one of the letters in terms of the other letter. You cannot solve a single linear equation that has two variables without some other restriction.

Please return to page $\frac{21}{2}$ and reconsider the question.

$\frac{6}{1}$

We do not agree. The expression -6n means -6 <u>times</u> a certain number or a second interpretation is "the negative of 6 times a number."

Please return to page $\frac{2}{1}$ and try this question again.

$\frac{6}{2}$

It appears that you made two mistakes.

On the one hand, you evidently thought that the perimeter equals the sum of the length and the width, when it actually is the sum of all the sides (in this case, it is the sum of two lengths and two widths).

Secondly, it seems that you gave as your answer the value of the width when you were asked for the value of the length.

Please return to page $\frac{22}{2}$ and be more careful in answering this problem.

$\frac{7}{1}$

Very good. You made the correct choice. If we let s represent the number of students in your class, then

$$(s + 1)$$

represents a class with one more student. Now, there are three such classes. Hence, the total number of students is

$$3 \quad (s + 1)$$

Please go on to question 4 below.

$\frac{7}{2}$

Question 4

Apply your knowledge and select the letter next to the correct answer to the question:

" If x represents p's age, and Q is 4 years less than 3 times p's age, then what expression would represent Q's age?"

(A) $3x - 4$

(B) $4x - 3$

(C) $3 (x - 4)$

(D) $\frac{1}{3} + 4x$

$\frac{8}{1}$

Did you take a guess? Please read the question at least twice until you have the facts clearly in your mind. Let us restate the conditions of this question. We start with the number 10 , increase it by some number , n . After this is done, we find that this sum equals 6 times the number n . Can you now write the equation?

Please return to page $\frac{29}{2}$ and try this question again.

$\frac{8}{2}$

The main reason why problem solving is so difficult in algebra is because it demands very accurate reading to extract the facts and relationships from the words.

It appears that you interpreted the problem to state that the perimeter was 17 inches. From this point, you went on to find one side and then calculated the value of the perimeter to be 17 inches more than the side.

Look, start over again. Read the problem, read the problem, and read the problem.

Please return to page $\frac{27}{2}$ and try this question again.

$\frac{9}{1}$

Do you know what the expression

$$12n + 2$$

means?

One of its translations is " 2 more than 12 times a number n ."

Please return to page $\frac{20}{2}$ and try this question again.

$\frac{9}{2}$

How many times did you read this problem?

You should read the problem once just to get acquainted with the facts. Then, you read it once more; this time to see how to represent the unknowns. There are two numbers involved here, a smaller and a larger one. The second sentence tells us that the difference between the two numbers is 12 . This is another way of saying that one number is 12 more than the other. You can, therefore, start by saying

let: x = the smaller number.

$x + 12$ = the larger number.

Now, read the problem again; this time try to write an equation.

Please return to page $\frac{11}{2}$ and try this question again.

10/1

What you chose is " 6 less a certain number."

The word than in the phrase " 6 less than a certain number, " makes a difference.

Please return to page 2/1 and try this question again.

10/2

The expression

$$3 (n - 4) = 8$$

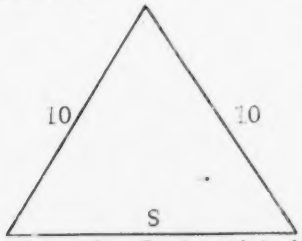
is incorrect. If we translate your choice back into English, it would read: When a number is decreased by 4 and then multiplied by 3 , the result equals 8 . Clearly, this is not the same as the given sentence. This would give precedence to the subtraction.

The English phrasing should follow the "order of operations" that we discussed earlier. Remember, multiplication takes precedence over addition or subtraction. Therefore, "three times a number" is represented by "3n" and then "decreased by 4 " is translated as "3n - 4 ."

Please return to page 18/2 and try this question again.

Very good. You made the correct choice.
We begin by drawing a diagram.

Part I: THE VARIABLES:



Let S = the length of the third side. Now the perimeter of the isosceles triangle is the sum of the lengths of all of its sides: that is,

$$10 + 10 + S = P$$

Part II: THE RELATIONSHIP:

We are told that this perimeter is 4 times the third side. Thus, we have the equation, $p = 4S$. We substitute $4S$ for p in our first equation.

$$10 + 10 + S = 4S$$

This equation in one variable can be solved.

Please go on to question 10 below.

Question 10

Apply your knowledge and solve the following problem:

The sum of two numbers is 4 times the smaller number. The difference of the two numbers is 12. What is the larger number?

(A) 18 (C) 8

(B) 16 (D) 6

$\frac{12}{1}$

The question should not only be read carefully, but it should be read more than once. Keep in mind that B is not quite 3 times as old as A .

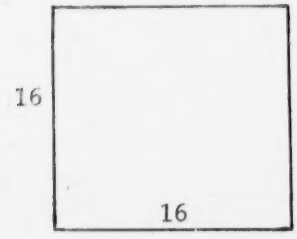
It is a good idea to list the various parts of the problem and to indicate what algebraic statement you have chosen for each part. For example:

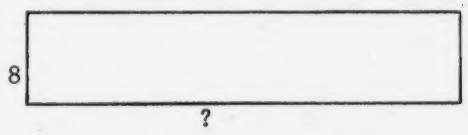
Let	P's age be:	x
Then	3 times P's age:	3x
Then	Q is 4 years less than 3 times P's age.	

Please return to page $\frac{7}{2}$ and try this question again.

$\frac{12}{2}$

We do not agree. Start by making a diagram. See below.





Now, answer the following questions. What is the area of the square? What is the area of the rectangle? What is the relation between the areas? What dimension of the rectangle should I denote by x ? What equation can I write.

Please return to page $\frac{30}{2}$ and try this question again.

The expression that you chose is incorrect. We want "twelve more than twice a number." Start with the last three words of the English phrase. Thus, if we let n be the number, twice that number is 2n.

Please continue.

Please return to page 20/2 and try this question again.

Did you do this problem too quickly?

You don't really think that the perimeter is "the sum of the length and the width" do you?

Isn't the perimeter the sum of ALL the sides?

Please return to page 22/2 and try this question again.

14/1

It is always a good idea to take a critical look at your answer and check to see if it really says what you intended.

Your choice states that 6 times a certain number n equals 10 .

Surely, you can see that this is not the same as the given sentence.

Please return to page 29/2 and try this question again.

14/2

We do not agree. Did you check your answer with the words of the problem? How can the bottle cost 10¢ (or more) when the liquid costs 23¢ more than the bottle.

Because then the liquid would have to cost 33¢ , and yet the total cost of both bottle and liquid is 33¢.

Please return to page 19/2 and try this question again.

$\frac{15}{1}$

You read the problem and extracted the facts correctly. Under the given conditions, the perimeter would turn out to be $25\frac{1}{2}$ inches , but let us ask you another question. Is that what you were asked to find?

Please return to page $\frac{27}{2}$ and re-read the question.

$\frac{15}{2}$

We do not agree. This is not an easy problem, but this is all the more reason why you will enjoy getting it. Listen to this.

If the boy works for only seven months instead of a full year, his reward should be only a fraction of the full year's reward. ($240.00 plus a horse.) Start by letting

x = the value of the horse.

Since it would detract from your pleasure of solving this problem yourself, no further help is being offered.

Please return to page $\frac{46}{2}$ and try this question again.

16/1

We don't agree. Try reading the given expression by starting with the last three words first. That is, "a certain number ____".

Please return to page 2/1 and try this question again.

16/1

We do not agree.

To get you started, note that the purchase consists of two kinds of stamps, some at 4¢ each and the rest at 8¢ each.

I: THE VARIABLES:

First, we consider the number of stamps.

Now, if we

let x = the number of 8¢ stamps to be purchased, then

3x = the number of 4¢ stamps to be purchased.

(" 3 times as many ")

II: THE RELATIONSHIP:

Second, we consider the value of each purchase. The question that you must ask yourself and answer is: How many pennies does Helen spend in buying x stamps at 8¢ each, and how many pennies does she spend in buying 3x stamps at 4¢ each? The total spent equals 100 pennies.

Please return to page 23/2 and try this question again.

Actually, this answer is not wrong for if you let the

width = x

then the length is

represented by x + 5 in this problem.

But a second relationship was given to you in the problem. You were told that the perimeter = 38 inches. This limits the possibilities of values for the length and width. The values x and x + 5 are "open" as they stand, but if they are related to a FIXED perimeter, it is possible to find the actual value of the dimensions in inches, the unit measure given in the problem.

Set up an equation.

Please return to page 22/2 and complete the problem.

We do not agree. If the boy did thirteen problems correctly, he would receive

13 x 8 = $1.04

If he did thirteen problems incorrectly, he would be fined

13 x 5 = $ .65

His father would owe him $.39.

Please return to page 36/2 and try this question again.

18
1

Yes, you made the correct choice.

The problem states that B is 4 years less than 3 times A's age. Again, it makes sense to read the English phrase starting with the last few words first. That is, B is 3 times A's age; but not quite less 4 years. Thus, 3 times A's age is represented by 3x and less 4 means -4 .

The answer is, therefore,

(3x - 4)

Please go on to question 5 below.

18
2 Question 5

Apply your knowledge and write an equation which corresponds to the conditions expressed by the sentence:

"Three times a number decreased by four equals eight."

Select the letter next to the correct answer.

(A) 3n + 8 = 4

(B) 3n - 4 = 8

(C) 3 (n - 4) = 8

(D) 3 (n - 4 = 8)

Very good. You made the correct choice.

I: THE VARIABLES:
We note that the difference between the two numbers is 12 . Hence,

let x = the smaller number

$x + 12$ = the larger number.

II: THE RELATIONSHIP:
The first sentence tells us that the sum of the two numbers, that is,

$$x + x + 12$$

is four times the smaller number. We therefore, can write the equation,

$$x + x + 12 = 4x$$

Combining like terms, we get

$$2x + 12 = 4x \quad \text{-2x}$$

$$12 = 2x \quad \div 2$$

$$6 = x$$

The smaller number is 6 , and the larger number is

$$x + 12 \quad \text{or} \quad 18$$

III: THE CHECK:
We now check these answers with the words of the problem.
The sum of the two numbers is four times the smaller number.

$$6 + 18 \stackrel{?}{=} 4(6)$$

$$24 = 24 \checkmark$$

Please go on to question 11 below.

Question 11

Apply your knowledge and solve the following problem:
A bottle of ginger ale costs 33¢ .
The liquid costs 23¢ more than the bottle.
What is the cost of the bottle? Select the letter next to the correct answer.

(A) 5¢

(B) 10¢

(C) More than 10¢ but less than 20¢ .

(D) More than 20¢ but less than 30¢ .

$\frac{20}{1}$

Yes, you made the correct choice. The expression " 6 less than a certain number " is translated in a special way. Let's try to justify it. We have a number in mind and denote it by the letter n . Now 6 less than that number is the number minus 6 ; that is, (n - 6) .

In algebra there are idioms (special ways of saying certain common ideas) just as there are in other languages.

Please go on to the next question below.

$\frac{20}{2}$

Question 2

Recognize the algebraic phrase which is a translation of the English phrase:

" Twelve more than twice a number. "

Select the letter next to the correct answer.

(A) $12 > 2n$

(B) $12n + 2$

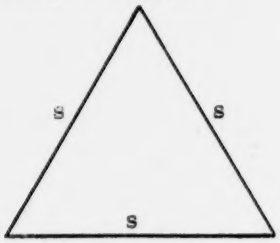
(C) $2 (12 + n)$

(D) $2n + 12$

Very good. You made the correct choice.

Part I: THE VARIABLES:

We start by making a diagram.



Now, an equilateral triangle is a triangle with all of its sides equal. Thus, each side can be represented by the same letter.

Let s = the measure of a side of the equilateral triangle.

then, $3s$ = the perimeter of the triangle.

Part II: THE RELATIONSHIP:

Reading the problem once again, we note that the perimeter, $3s$, is equal to 17 inches more than the length of one side. We, therefore, have the equation

$$3s = s + 17$$

Subtracting s from both members of this equation, we get

$$2s = 17 \quad \text{and}$$

$$s = 8\frac{1}{2}$$

Please go on to question 9 below.

Question 9

Apply the principles of problem solving and choose the single equation that is sufficient to lead directly to the solution of the following problem:

" Each of the two equal sides of an isosceles triangle is 10 inches in length. The perimeter is four times as long as the third side. Find the third side. "

Select the letter next to the correct equation:

(A) $P = 4s$

(B) $P = 10 + 10 + s$

(C) $4s = 10 + 10 + s$

(D) $10 + 10 + x = 4s$

$\frac{22}{1}$

Very good. You made the correct choice. We start with the number 10 , increase it by some number n . We now have

$$10 + n$$

We are told that this sum equals 6 times the number n . Hence, we get the equation,

$$10 + n = 6n$$

Please go on to question 7 below.

$\frac{22}{2}$

Question 7

Apply your knowledge and solve the following problem:

" The perimeter of a rectangle is 38 inches.
The measure of the length is 5 inches greater than the measure of the width.

What are the measures of the length? "

Select the letter next to the correct answer.

(A) the length is $16\frac{1}{2}$ inches

(B) the length is $21\frac{1}{2}$ inches

(C) the length is 12 inches

(D) the length is $x + 5$ inches

Very good. You made the correct choice.

I: THE VARIABLES:

A square whose sides are 16 inches has an area equal to

$$16 \times 16 = 256 \text{ square inches}$$

Since the problem asks for the length of the rectangle, we let

$$x = \text{that length}$$

II: THE RELATIONSHIP:

Now, the area of a rectangle is the product of its length and width; that is $8x$. We, therefore, have the equation,

$$8x = 256$$

Dividing both members of the above equation by 8 we get

$$x = 32$$

III: THE CHECK:

area of square	=	area of rectangle
16×16	$\stackrel{?}{=}$	8×32
256	$=$	256 ✓

Please go on to question 13 below.

Question 13

Apply your knowledge and solve the following problem: Helen wants to buy three times as many 4¢ stamps as 8¢ stamps. How many stamps of each type can she buy for one dollar?

Select the letter next to the correct answer.

(A) 8 more 4¢ stamps than 8¢ stamps.

(B) four 8¢ stamps, twelve at 4¢.

(C) 10 stamps more at 4¢ than at 8¢.

(D) 25 stamps in all.

$\frac{24}{1}$

There is a natural phrasing in English that cannot be overlooked when you translate to algebra. Although the words were translated by you in the same order that they appeared, your algebra does not say what the English said. It does not mean " Three times the equation, 'a number decreased by four equals eight' . "

Return to page $\frac{18}{2}$ and try this question again.

$\frac{24}{2}$

Are you sure you didn't find the cost of the liquid?

You were asked for the cost of the bottle.

Return to page $\frac{19}{2}$ and reconsider.

Yes, your choice is correct.

Algebraic phrases are not always stated in the same order as their English phrases. Thus, to express the given English phrase algebraically, it sometimes helps to restate it in different words.

Let's try this way. We want a number which is as big as the result of doubling a certain number and then adding twelve to the answer.

Twice a number is $2n$ if n is the number.

Twelve more than this is

$$2n + 12$$

Please go on to question 3 below.

Question 3

Choose a variable to represent the number of students in your class, and write an algebraic phrase that will represent the total number of students in three other classes if each of the other classes has one more student than your class.

Select the letter next to the correct answer.

(A) $3s + 1$ (C) $4s$

(B) $3(s + 1)$ (D) None of these.

$\frac{26}{1}$

We do not agree. Let us help you with Part I .

Note that twelve bars of candy were purchased in all.

If we let x = the number of 6¢ bars purchased, then

(12 - x) = the number of 10¢ bars purchased.

Keep this in mind: If the sum of two quantities is given, and one is denoted by x , the other quantity is the given sum minus x .

Now continue with Part II where you must establish a relation between the two amounts listed in Part I .

Be sure to check your answers.

Please return to page $\frac{38}{2}$ and try this question again.

$\frac{26}{2}$

We do not agree. A quick check will prove the point.

If he washed the car three times, he would earn \$3 , and if he mowed the lawn three times, he would earn

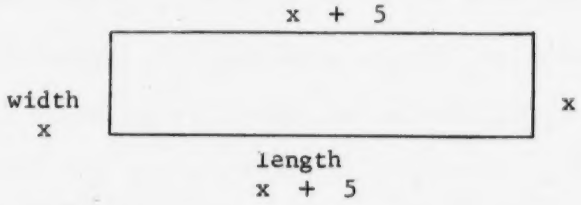
3 (1.50) = \$4.50

That is \$7.50 altogether.

Your answer does not check.

Please return to page $\frac{34}{2}$ and try this question again.

Very good. You made the correct choice. It is always a good idea to draw the figure of the problem.



Part I: THE VARIABLES:
Since the length of the rectangle is expressed in terms of its width, we will first

let x = the width

and then $x + 5$ would equal the length.

Now the perimeter of a rectangle is the sum of the measures of all of its sides. Thus,

width + length + width + length = perimeter

$x + (x + 5) + x + (x + 5) = 4x + 10$

Part II: THE RELATIONSHIP:
We are told that the perimeter is equal to 38 . Hence, we have the equation,

$4x + 10 = 38$ $\not{-}\ 10$

$4x = 28$ $\not{\div}\ 4$

$x = 7$

Thus, the width is 7 and the length, which is 5 more, is 12 .

Part III: THE CHECK:
Finally to make sure, we check these answers with the original problem. The dimensions are

$7 + 12 + 7 + 12 = 38$

$38 = 38$

Please go on to question 8 below.

Question 8

Apply your knowledge and solve the following problem: The perimeter of an equilateral triangle is 17 inches more than the length of one side; find the length of a side.

(A) $5\frac{2}{3}$ inches (C) $22\frac{2}{3}$ inches

(B) $8\frac{1}{2}$ inches (D) $25\frac{1}{2}$ inches

$\frac{28}{1}$

If you took the trouble to check the answer you chose, you would have discovered that your choice is incorrect. A purchase of four stamps at 8¢ each amoutns to 32¢ . A purchase of twelve stamps at 4¢ each amounts to 48¢ . The sum of these two purchases is 80¢ and not one dollar.

Return to page $\frac{23}{2}$ and try this question again.

$\frac{28}{2}$

We do not agree. The question asks " how many years ago. "

Therefore, let

x = the number of years ago

Since the present ages of father and son are 36 and 12 respectively, x years ago, their ages were

$36 - x$ and $12 - x$

That is, each one was x years less (younger).

Please continue.

Return to page $\frac{44}{2}$ and try this question again.

Very good. You made the correct choice.

First, we replace the word"number"by some letter, say," n ".

We read the problem again and as we go on, we change the English words into algebraic expressions. Thus,"three times a number"becomes "3n" ; "decreased by 4" becomes

$$3n - 4$$

and finally"equals 8" becomes

$$3n - 4 = 8$$

Please go on to the next question below.

Question 6

Apply your knowledge and write an equation which corresponds to the conditions expressed by the sentence: " Ten increased by a number equals six times the same number. "

Select the letter next to the correct answer.

(A) $6n + 10 = n$

(B) $10n = 6 \times n$

(C) $10 + n = 6n$

(D) $6n = 10$

30
1

Very good. You made the correct choice.

I: THE VARIABLES:
Note that the cost of the liquid is compared with the cost of the bottle. We therefore,

let x = the cost of the bottle, and

$x + 23$ = the cost of the liquid.

II: THE RELATIONSHIP:
Since the cost of both the bottle and liquid is 33¢ , we can write the equation,

$$x + x + 23 = 33 \quad [\text{ Collect}$$
$$2x + 23 = 33 \quad \swarrow - 23$$
$$2x = 10 \quad \swarrow \div 2$$
$$x = 5$$

Thus, the bottle costs 5¢ and the liquid costs $5 + 23 = 28$¢ .

III: THE CHECK:
Note that the sum of the separate items equals the total cost

$$5 + 28 = 33$$
$$33 = 33 \checkmark$$

so that the answer checks.

Please go on to question 12 below.

30
2

Question 12

Apply your knowledge and solve the following problem: A square whose sides are each 16 inches long has the same area as a rectangle whose width is 8 inches. Find the length of this rectangle. Select the letter next to the correct answer.

(A) 24 inches (C) 42 inches

(B) 36 inches (D) 32 inches

We do not agree. One of the letters does have the correct answer next to it. Please reconsider your choice. Set up your values in a chart form.

Read carefully, as we've said many times before.

Return to page 38/2 and try this question again.

You cannot be blamed for your choice. Sometimes a problem in algebra does sound like we are asking for the impossible. However, your choice is incorrect; the answer can be determined, and one of the letters does have the correct answer next to it.

Return to page 46/2 and try this question again.

$\frac{32}{1}$

Ten cents seems to be the obvious answer, but a little reflection on your part will make you realize that 10¢ is incorrect.

If the bottle costs 10¢ the lotion must cost $1 , but $1 is only 90¢ more than 10¢ .

Do you see the mistake? Now attack the problem with an algebraic method.

Return to page $\frac{48}{2}$ and try this question again.

$\frac{32}{2}$

Your choice is incorrect. Since the boy receives more money for doing a problem correctly than he is fined for doing one incorrectly, the boy has to do <u>less</u> than half the problems correctly to come out even.

Suppose you stop guessing and do the problem algebraically.

Let x = the number of problems the boy did correctly

and $26 - x$ = the number of problems he did incorrectly.

Note that the amount earned is equal to the amount that the boy is fined.

Return to page $\frac{36}{2}$ and try this question again.

$\frac{33}{1}$

We do not agree. If 25 stamps were purchased for one dollar, then only 4¢ stamps could be purchased since

$$25 \times 4 = 100$$

A common sense check can be helpful in discovering a wrong answer. However, you need algebra to help you get the right answer. Start by letting

x = the number of 8¢ stamps purchased, and

$3x$ = the number of 4¢ stamps purchased.

Now if you purchase one stamp at 4¢ you spend 4¢ ; if you purchase two stamps at 4¢ each , you spend 8¢ ; and if you purchase x stamps at 4¢ each, you spend x times 4¢ or $4x$ cents.

Please continue.

Please return to page $\frac{23}{2}$ and try this question again.

$\frac{33}{2}$

We do not agree. The set of integers is a larger set than the set of natural numbers , but it does not include fractions.

Please return to page $\frac{47}{2}$ and try this question again.

$\frac{34}{1}$

Very good. You made the correct choice.

I: THE VARIABLES:

We start by letting

x = the number of years ago

$36 - x$ = the father's age x years ago

$12 - x$ = the son's age x years ago.

II: THE RELATIONSHIP:

Now, the father was four times as old as his son was then. Hence, we have the equation,

$$\begin{aligned} 36 - x &= 4(12 - x) && [\text{D} \\ 36 - x &= 48 - 4x && \swarrow +4x \\ 36 + 3x &= 48 && \swarrow -36 \\ 3x &= 12 && \swarrow \div 3 \\ x &= 4 \end{aligned}$$

III: THE CHECK:

Let us check these answers with the words of the problem.

Four years ago the father was $36 - 4 = 32$ years old

the son was $12 - 4 = 8$ years old

and 32 equals four times 8 .

These relationships can be arranged in a compact chart form:

	Age Now	Age x years ago	Check	
Father	36	$36 - x$	36	32
Son	12	$12 - x$	12	8
	$(36 - x) = 4(12 - x)$		$32 = 4(8)$ ✓	

Please go on to question 16 below.

$\frac{34}{2}$

Question 16

Apply your knowledge and solve the following problem:

Herbert receives one dollar for washing the car and $1\frac{1}{2}$ dollars for mowing the lawn. During one month, he earned seven dollars for doing six jobs. How many times did he wash the car and mow the lawn?

Select the letter next to the correct answer.

(A) 3 and 3 (C) 4 and 2

(B) 5 and 1 (D) It is possible to have more than one answer.

$\frac{35}{1}$

Did you find this problem a little confusing? There are three boys involved and to avoid confusion, you should read the problem a few times. Notice, and this is important, that everybody's performance is compared with Peter's. When a number of quantities are each compared with one particular quantity, denote the particular quantity by x. Thus, in this problem,

let x = the number of papers that Peter delivers.

Please continue.

Please return to page $\frac{41}{2}$ and try this question again.

$\frac{35}{2}$

Consecutive means "following in order."

For example, the integers 3, 4, 5, 6, are consecutive.

4 follows 3; 5 follows 4, etc.

Please return to page $\frac{53}{2}$ and try this question again.

$\frac{36}{1}$

Very good. You made the correct choice.

I: THE VARIABLES:

We begin by letting

x = the value of the horse

Now the boy worked for seven months and is, therefore, entitled to only $\frac{7}{12}$ of the original agreement; that is

$$\frac{7}{12}\ (240 + x)$$

II: THE RELATIONSHIP:

This equals what the boy actually received. We thus have the equation,

$$\frac{7}{12}\ (240 + x) = 100 + x \quad \times 12$$

$$7\ (240 + x) = 12\ (100 + x) \quad [\ D$$

$$1680 + 7x = 1200 + 12x \quad -1200$$

$$480 + 7x = 12x \quad -7x$$

$$480 = 5x \quad \div 5$$

$$96 = x$$

Please go on to the next question.

$\frac{36}{2}$

Question 20

Apply your knowledge and solve the following problem:

A father offered to pay his son 8¢ for every problem that he solved correctly, and fine him 5¢ for every problem he did incorrectly. The boy did 26 problems and neither the father nor the son owed any money to each other. How many problems did the boy solve correctly? Select the letter next to the correct answer.

(A) 13 problems

(B) More than half of them

(C) More than 8 , but less than 13

(D) It is not fair.

That's not the usual guess. Did you check it?

Now do the problem algebraically; you will find it more reliable.

Start by letting

x = the cost of the bottle, and

$100 + x$ = the cost of the lotion.

Please continue.

Please return to page $\frac{48}{2}$ and try this question again.

One of the ways to analyze a statement in algebra is to apply it to a few specific cases. This usually focuses your attention on the relationships involved. Therefore, suppose you replace the letters r , s , t , by any three consecutive integers. For example,

let $r = 7$

$s = 8$

$t = 9$

Now, check all the choices and discover the correct one.

Please return to page $\frac{45}{2}$ and try this question again.

38
1

Yes, your choice is correct. You can proceed in the following way:

I: Numbers:

Let

x = the number of stamps bought at 8¢ each.

let $3x$ = the number of stamps bought at 4¢ each. Then,

II: Value:

$8x$ = the cost in pennies of the first purchase.

$3x\ (4) = 12x$ is the cost in pennies of the second purchase.

III: Total Cost:

Both purchases

$8x + 12x = 100$ Combining like terms, we get

$20x = 100$

$x = 5$ and

$3x = 15$

Thus, Helen bought five stamps at 8¢ each and fifteen stamps at 4¢ each.

IV: Check:

Note that:

$5 \times 8 = 40$

$15 \times 4 = 60$ and

$40 + 60 = 100$ ✓

This information can be organized into a chart:

No. of stamps		Value of each	Total value in cents	Check:		
(a)	x	8¢	$8x$	5	8	40
(b)	$3x$	4¢	$(3x)\ (4)$	15	4	60
both		$8x + 12x =$	100	100		100

Please go on to question 14 below.

38
2

Question 14

Apply your knowledge and solve the following problem: One bar of candy costs 6¢ another bar costs 10¢ . A dozen bars were purchased for one dollar. Select the letter next to the correct statement about the purchase.

(A) 10 bars @ 6¢ were bought.

(B) 7 @ 10¢ were bought.

(C) Twice as many were bought for 6¢ as were bought for 10¢.

(D) None of the answers above is correct.

If Herbert washed the car five times he would earn \$5 . If he mowed the lawn once, he would earn \$1.50 and this adds up to \$6.50 . Thus, your answer does not check with the conditions of the problem. Please note that six jobs were done. If we

x = the number of times that Herbert washed the car, then

$6 - x$ = the number of times that he mowed the lawn.

Next, consider the reward for each job. Herbert earns one dollar for one car wash; he earns x times as much for x car washes.

Please continue.

Please return to page 34/2 and try this question again.

An even number is a number that is exactly divisible by 2 . Thus, the set of even counting numbers is

$$\{2, 4, 6, 8, \ldots\}$$

Note that even numbers can be obtained by starting with an even number and counting by twos.

Please return to page 56/2 and try this question again.

$\frac{40}{1}$

You said "never." You probably reasoned that presently the father is three times as old as his son, and this condition will never change. Your reasoning is wrong. It is true that two people will always remain the same number of years apart, since both get older at the same rate. However, the ratio of their ages does change. If you are now 15 years old and you have a brother that is 3 years old, you are now five times as old as he is. But in three years, you will be 18 and your brother will be 6 . Thus, you will be only three times as old as he is. You will always be twelve years <u>older</u> than your brother but the ratio of your ages will change.

Please return to page $\frac{44}{2}$ and try this question again.

$\frac{40}{2}$

Let's point out your error indirectly by giving you two examples of the principle involved.

If x is an integer, then

$$(x + 1)$$

is the next consecutive integer.

If $2y + 7$ is an integer, than $(2y + 7) + 1$ is the next consecutive integer.

Please return to page $\frac{5}{2}$ and try this question again.

Very good. You made the correct choice.

I: THE VARIABLES:

We start by letting

$$x = \text{the cost of the bottle}$$

$$x + 100 = \text{the cost of the lotion.}$$

II: THE RELATIONSHIP:

Now,

$$x + x + 100$$

is the total cost. Hence, we have the equation,

$$2x + 100 = 110 \quad \text{<-100}$$

$$2x = 10 \quad \text{<÷2}$$

$$x = 5$$

Thus, the bottle costs 5¢ and the lotion costs $1.05 .

Please go on to question 18 below.

Question 18

Apply your knowledge and solve the following problem:

Peter, Mike, and Chuck deliver 256 Sunday papers.

Mike delivers 28 more papers than Peter.

Chuck delivers 12 papers less than twice as many as Peter.

How many papers does each one deliver. Select the letter next to the correct answer.

(A) Peter 60 , Mike 108 , Chuck 88

(B) Peter 108 , Mike 60 , Chuck 88

(C) Peter 88 , Mike 60 , Chuck 108

(D) None of these.

$\frac{42}{1}$

It is easy to agree with you. You feel that if a boy did 26 problems right or wrong, he is entitled to some reward. However, this is not a matter of feeling; it is simply a matter of living up to a specific arrangement agreed upon by both parties.

Please return to page $\frac{36}{2}$ and try this problem from an algebraic point of view.

$\frac{42}{2}$

Let us write the set of counting numbers

$$\{1, 2, 3, 4, 5, 6, 7, 8, \ldots$$

Note that the integers alternate; odd, even, odd, even, and so on.
Thus, if we start with an odd integer, the consecutive odd integer is obtained by adding 2 to the integer we started with. How is the next <u>smaller</u> integer obtained?

Please return to page $\frac{62}{2}$ and try this question again.

We don't agree. One of the other letters does have the correct answer next to it. But did you find more than one answer to the problem before you made that statement?

Please return to page 34/2 and try this question again.

We do not agree.

How do we represent three consecutive integers?

We let

x = the first integer,

$x + 1$ = the next integer,

$x + 2$ = the third integer.

Read the question again. This time note what we are told about these three integers.

Please return to page 72/2 and try this question again.

44
1

Very good. You made the correct choice. We do not know how many bars of candy of each kind were purchased, but we do know that a dozen bars were purchased in all.

I: THE VARIABLES:
If we let
x = the number of 6¢ bars purchased.
then $12-x$ = the number of 10¢ bars purchased.

Note that if the sum of two numbers is given and one of the numbers is denoted by x, the other number is denoted by the given sum minus x

II: THE RELATIONSHIP:
Now,
(a) the value in pennies of x bars at 6¢ each is $6x$, and
(b) the value in pennies of $(12 - x)$ bars at 10¢ each is $10(12 - x)$

We can, therefore, write the equation,
$$6x + 10(12 - x) = 100$$
since the total value of the purchase is \$1 or 100¢
Clearing parentheses, we get
$$6x + 120 - 10x = 100$$
Combining like terms, we obtain
$$-4x + 120 = 100$$
Subtracting 120 from both members, we get
$$-4x = -20$$
Using the fact that if two quantities are equal, their opposites are equal, the above equation becomes
$$4x = 20 \quad \text{and} \quad x = 5, \quad 12 - x = 7$$

III: THE CHECK:
This information can be compactly arranged as follows:

	No. of bars:	Value in ¢	Total Value in ¢		Check	
(a)	x	6	$6x$	5	6	30
(b)	$12 - x$	10	$10(12 - x)$	7	10	70
			$6x + 10(12 - x) = 100$			100 ✓

Please go on to question 15 below.

44
2

Apply your knowledge and solve the following problem:
A father is 36 years old and his son is 12 years old. A few years ago, the father was four times as old as his son was then.
How many years ago was this?
Select the letter next to the correct answer.

(A) 3 years ago
(B) 6 years ago
(C) More than 3 years ago but less than 6 years ago.
(D) Never

$\frac{45}{1}$

Yes, your choice is correct. Consecutive integers are obtained by counting by ones from smaller to larger.

e. g.

$$1 , \ 2 , \ 3 , \ 4 , \ \ldots$$

Thus, the next consecutive integer to $k + 5$ is

$$k + 5 + 1 = k + 6$$

Please go on to question 3 below.

$\frac{45}{2}$

Question 3

Apply your knowledge and select the letter which completes the following statement correctly?

If r, s, t are three consecutive integers, then_____".

(A) $t = r + s$

(B) $s = t - r$

(C) $t = r + 2$

(D) $r = t - 1$

46
1

Very good. You made the correct choice. It took courage on your part to say " none of these. " Did you notice that both Mike's and Chuck's performance are compared with Peter's.

I: THE VARIABLES:

We let x = the number of papers that Peter delivers

$2x + 2$ = the number of papers that Mike delivers

$x + 28$ = the number of papers that Chuck delivers.

II: THE RELATIONSHIP:

The three boys together deliver 256 papers. We, therefore, have the equation,

$$(x) + (x + 28) + (2x + 2) = 256 \quad [\text{C}$$

$$4x + 16 = 256 \quad \text{<- } 16$$

$$4x = 240 \quad \text{<÷ } 4$$

$$x = 60$$

Hence, Peter delivers 60 papers, Mike delivers 88 , and Chuck 108 .

Please go on to question 19 below.

46
2

Question 19

Apply your knowledge and solve the following problem: A boy agreed to work one year for $240 and a horse. At the end of seven months, he quit and received $100 and the horse. What was the value of the horse? Select the letter next to the correct answer.

(A) $168 (C) $120

(B) $ 96 (D) Cannot be determined.

VOLUME 6 SEGMENT 2 begins here:

Obtain a PUNCH CARD form your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	2 7	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 6	(Volume Number)
	66	and	68	0 2	(Segment Number)

Your READING ASSIGNMENT for this segment is pages 170 - 171 and 172 - 175 .

SUPPLEMENTARY NOTE:

In the previous segment, you were introduced to the algebraic solution of verbal problems. This segment is a continuation of the same work. You will be asked to solve additional types of verbal problems. You will be impressed with the great variety of problems that can be solved by algebraic methods; that is, by writing an equation that represents the conditions expressed in words.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Recognize and select the letter next to the set(s) whose elements are integers.

$$S = \left\{1 , 2 , 3 , \frac{1}{2} , \frac{1}{3}\right\}$$

$$T = \left\{-5 , -4 , -3 , -2 , -1 , 0\right\}$$

$$U = \left\{1 , -1 , \frac{1}{2} , \frac{1}{2}\right\}$$

$$V = \left\{0 , 2 , 4 , 6 , 8 , 10\right\}$$

(A) set S and set U

(B) only set V

(C) sets T and V

(D) only set T

48
1

Very good. You made the correct choice.

I: THE VARIABLES:
Since Herbert performed six jobs,

let x = the number of times that he washed the car,

$6 - x$ = the number of times that he mowed the lawn.

Note that if the sum of two numbers is given, and one of these numbers is denoted by x, then the other number is denoted by the given sum minus x.

II: THE RELATIONSHIP:
Now, the amount of money (in pennies) earned by x car washes is $100x$, and the amount of money earned by $(6 - x)$ mowings is

$$150 (6 - x)$$

We, therefore, have the equation,

$$100x + 150 (6 - x) = 700 \quad [\text{ D}$$

$$100x + 900 - 150x = 700 \quad [\text{ C}$$

$$-50x + 900 = 700 \quad \swarrow -900$$

$$-50x = -200 \quad \swarrow \div (-50)$$

$$x = 4$$

III: THE CHECK:
Finally, we check these answers with the words of the problem:

4	jobs	@	$1.00	= $4.00
2	jobs	@	$1.50	= $3.00
	total			= $7.00 ✓

Please go on to question 17 below.

48
2

Question 17

Here's an old " puzzler. "
Apply your knowledge and solve the following problem:

A bottle of hand lotion costs $1.10. If the hand lotion costs $1 more than the bottle, how much does the bottle cost?

Select the letter next to the correct answer by estimating the answer.

(A) 10¢

(B) More than 6¢ but less than 10¢

(C) $2\frac{1}{2}$ ¢

(D) Less than 6¢

Very good. Your choice is correct.

I: THE VARIABLE:

We start by letting

x = the number of problems the boy did correctly;

$26 - x$ = the number of problems the boy did incorrectly.

Now, for every correct problem the boy earns 8¢ ; hence, for x correct problems, he earns $8x$ cents. For every incorrect problem, he is fined 5¢ and for $(26 - x)$ such problems he is fined

$$5 (26 - x)$$

II: THE RELATIONSHIP:

Since both father and son are even, the amount earned by the boy and the fines due to the father are equal. We, therefore, have the equation,

$$8x = 5 (26 - x) \qquad [\text{D}$$

$$8x = 130 - 5x \qquad + 5x$$

$$13x = 130 \qquad \div 13$$

$$x = 10$$

The boy did 10 problems correctly and received

$$10 \times 8¢ = 80¢$$

He did 16 problems incorrectly and paid his father

$$16 \times 5¢ = 80¢$$

They came out even.

You have now finished this segment. Hand in your PUNCH CARD. You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT: Problems 1 , 2 , 3 , and 4 .

$\frac{50}{1}$

Keep in mind that consecutive means "follows in order."

Thus, for example, if x is an integer,

$$x + 1 \text{ and } x + 2$$

are two consecutive integers.

What is the relationship between the first integer and the third?

Please return to page $\frac{45}{2}$ and try this question again.

$\frac{50}{2}$

Did you check your answers?

If the largest integer is 30 then the three consecutive integers are

$$28 , 29 , 30$$

Please find the sum of these three integers.

Please return to page $\frac{72}{2}$ and try this question again.

Yes, your choice is correct.

If we start with an odd integer, the next smaller odd integer is obtained by subtracting 2 .

Thus,

$$(2n + 1) - 2 =$$

$$2n - 1$$

is the next smaller integer.

Please go on to the next question.

Question 6

Apply your knowledge and solve the following problem:

"The sum of two consecutive integers is 37 ; find the numbers."

Select the letter next to the correct answer.

(A) The smaller number is 14

(B) One of the numbers is 19

(C) The larger is 18

(D) There aren't two such consecutive integers

$\frac{52}{1}$

We do not agree. If we start with an even number, say 12 , what is the next even number?

Yes, 14 is the answer. That is, if we start with an even number, the next even number is obtained by <u>adding 2</u> . (The odd and even numbers alternate.)

Note: The expression, $n - 3$ can represent either an odd or an even number, depending upon the value given to n . In this problem it was given that the expression represents an even number. Don't let that -3 fool you.

Return to page $\frac{56}{2}$ and try this question again.

$\frac{52}{2}$

We do not agree. Please note that this question deals with consecutive <u>even</u> integers. If we start with an even integer, 2 must be added to obtain the next even integer. Thus the three consecutive even integers should be represented by

$$x \ , \ x + 2 \ , \ x + 4$$

Return to page $\frac{60}{2}$ and try this question again.

Very good. You made the correct choice. The set of integers consists of the union of the set of natural numbers, their opposites and zero. Thus, the set of integers is,

$$I = \{\ldots\ -4\ ,\ -3\ ,\ -2\ ,\ -1\ ,\ 0\ ,\ 1\ ,\ 2\ ,\ 3\ ,\ 4\ \ \ldots\}$$

Please go on to question 2 below.

Question 2

Apply your knowledge and select the letter which answers the following question correctly:

" If $k + 5$ is an integer, what represents the next consecutive integer? "

(A) $2k + 5$

(B) $k + 6$

(C) $2k + 6$

(D) $k + 4$

$\frac{54}{1}$

We do not agree. It is true that one of the integers is exactly divisible by 3 , but it is not the largest of the three integers. Start as follows:

let x = the first odd integer

$x + 2$ = the next odd integer

$x + 4$ = the third odd integer

Just like with even integers, the next larger odd integer is obtained by adding 2 to the previous integer.

Return to page $\frac{70}{2}$ and try this question again.

$\frac{54}{2}$

You chose the answer that many people choose without thinking the problem through.

While the minute hand is moving around the clock doesn't the hour hand move a bit too? In order to be precise about it, you must determine just what fraction of the circle is the arc involved. There are $360°$ in the complete arc of the circle.

How many degrees would there be between two adjacent numbers indicating the hours?

Return to page $\frac{73}{2}$ and reconsider this problem.

$\frac{55}{1}$

We do not agree. The middle integer is divisible by 4 , but it is not divisible by 8 . Perhaps you misinterpreted the original problem.

Start by letting

x = the first integer

$x + 1$ = the consecutive number

$x + 2$ = the third consecutive number

Please return to page $\frac{72}{2}$ and try this question again.

$\frac{55}{2}$

Every angle of your choice is incorrect. Keep in mind the definition of complementary angles. Two angles are said to be complementary if their sum is $90°$. Thus, in the choice you made:

	Given Angle	Your Choice	The Sum	
I	$77°$	23	100	$\neq 90°$
II	$-110°$	80	-30	$\neq 90°$
III	$3d$	$90 + 3d$	$90 + 6d$	$\neq 90°$

$\frac{56}{1}$

Very good. You made the correct choice.

If r , s , t are consecutive integers, then s is one more than r , and t is one more that s .

That is, $t = r + 2$

Another way of stating this is to say that the difference between the first and the third consecutive integer is 2 .

Please go on to question 4 below.

$\frac{56}{2}$

Apply your knowledge and select the letter which correctly completes the following statement:

" If (n - 3) is the first of three consecutive even integers, then the last integer is represented by _____. "

(A) $n - 1$

(B) $n + 1$

(C) $n - 5$

(D) $n + 5$

If the smaller of the two consecutive integers is 14 , the larger is 15 .

Now,

$$14 + 15 = 29 \quad \text{and not} \quad 17$$

Start this problem by letting

$$x = \text{the smaller integer}$$

and

$$x + 1 = \text{the larger integer}$$

Return to page 51/2 and try this question again.

This problem requires careful reading. Since we are dealing with three consecutive odd integers, we represent them as follows:

Let x = the first odd integer

$x + 2$ = the second

$x + 4$ = the third

At this point you should read the problem again, phrase by phrase, and translate the English words into algebraic symbols. Thus, for example, " three times the sum of the first two ..., " can be written as

$$3 (x + x + 2)$$

Please continue and check your answers with the words of the problem.

Return to page 77/2 and try this question again.

58/1

Two angles are said to be supplementary when the sum of their degree measure is 180°

Now, $-80° + (-100)° = -180°$

Although the other examples in this choice are also wrong, you realize that one error is enough to throw out your choice as not being correct.

Return to page 79/2 and try this question again.

58/2

Keep in mind that a counter-clockwise rotation generates a positive angle, and a clockwise rotation generates a negative angle. Your text book explains this in great detail on page 173 . Please re-read this page.

First, find the value for each partial rotation and assign it to the x and y in the given expression.

Return to page 80/2 and try this question again.

You made a hasty choice.

If you examine the numbers that you selected, you will notice that they are not consecutive odd integers.

Please return to page 70/2 and try this question again.

No, the angle made by the hands of a clock is zero degrees at 12 o'clock, because they are both pointing in the same direction. Zero degrees occurs when both hands coincide. At 12:30 are they pointing exactly in opposite directions?

Please return to page 73/2 and try this question again.

60
1

Yes, your choice is correct.

I: THE VARIABLES:
We start as follows:

Let x = the first integer

$x + 1$ = the next integer

$x + 2$ = the third integer.

II: THE RELATIONSHIP:
Now, the sum of these three numbers is 84 . We, therefore, have the equation:

$$x + (x + 1) + (x + 2) = 84 \qquad [\ C \wedge A$$

$$3x + 3 = 84 \qquad \propto -3$$

$$3x = 81 \qquad \propto \div 3$$

$$x = 27$$

$$x + 1 = 28$$

$$x + 2 = 29$$

III: THE CHECK:
Therefore,

(a) The integers are consecutive:

27 , 28 , and 29 are consecutive integers.

(b) Their sum is 84 .

$27 + 28 + 29 = 84$ ✓

Please go on to question 8 below.

60
1

Question 8

Apply your knowledge and solve the following problem:

" Find three consecutive even integers whose sum is 102 . "

Select the letter next to the correct answer.

(A) The largest of the integers is divisible by 12 .

(B) The sum of the first two integers is 70 .

(C) The integers are 33 , 34 , 35

(D) The largest of the integers is not divisible by twelve

$\frac{61}{1}$

You made a hasty choice. If the larger is 18 , then the smaller is 17 .

Note that

$$17 + 18 = 35 \text{ , not } 37$$

Start by letting

$$x = \text{the smaller integer}$$

and

$$x + 1 = \text{the consecutive integer}$$

Please return to page $\frac{51}{2}$ and try this question again.

$\frac{61}{2}$

In order for two angles to be complementary, their sum must equal $90°$; that is, $+ 90°$.

When you add the given angle, $-110°$ to your choice, $20°$, you get $-90°$.

That is close, but not correct.

Please return to page $\frac{69}{1}$ and try this question again.

62/1

Very good. You made the correct choice.

If we start with an even number, the consecutive even number is obtained by adding 2 . Thus, if

$$n - 3$$

is an even number, the consecutive even number is

$$(n - 3) + 2 = n - 1$$

The even number after this one is

$$(n - 1) + 2 = n + 1$$

Note: since n is a variable, n - 3 could represent an odd or an even number depending upon the value given to n .
The -3 itself does not make it odd.

Please go on to question 5 below.

62/2

Apply your knowledge and select the letter which correctly completes the following statement:

" If (2n + 1) represents an odd integer, the next smaller odd integer is represented by _____. "

(A) 2n

(B) 2n - 3

(C) 2n - 1

(D) n - 1

$\frac{63}{1}$

The word supplement is defined in your reading assignment:

"One angle is said to be the supplement of a second angle if the sum of the degree measure of these angles is 180 . "

Now,

$$-80° + 100° = 20°$$

and not $180°$. Is it possible that at this late stage in algebra, you made a mistake in signs?

Please return to page $\frac{79}{2}$ and try this question again.

$\frac{63}{2}$

Your choice is interesting. It does indicate that you recognize the relationship "one angle is the complement of another." But there are two comments that can be made.

(1) Where does your choice take into consideration the relationship, "one angle is ten degrees more than its complement?" You correctly labelled the variables, but you ignored the second half of the problem, the "relationship."

(2) If you examine your choice you will see that it is an identity; that is, it has an unlimited number of members of the solution set. Any positive angle less than 90 will certainly fit the open sentence.

Please return to page $\frac{82}{2}$ and reconsider the problem.

$\frac{64}{1}$

We do not agree. One of the letters does have the correct answer next to it. In other words, there are such integers. You can determine them by letting

x = the first odd integer

$x + 2$ = the second odd integer

and $x + 4$ = the third odd integer

Now, re-read the problem to find the relationship that determines the equation involving these.

Please return to page $\frac{70}{2}$ and try this question again.

$\frac{64}{2}$

We don't agree. Make a picture of a clock. Now, there are 360° in a complete revolution (circle). The clock has twelve numbers marked on it, and these numbers divide the clock into twelve equal arcs. Answer this question:

"Through how many degrees does the minute hand move in five minutes?"

Please return to page $\frac{73}{2}$ and try this question again.

$\frac{65}{1}$

You found three consecutive integers whose sum is 102 . But a careful reading of the original problem will reveal that there was an added condition, the numbers were to be consecutive <u>even</u> numbers.

Once again, we must remind you to read very carefully (read and re-read).

Please return to page $\frac{60}{2}$ and do this problem correctly.

$\frac{65}{2}$

This problem requires careful reading. For Part I , The Variables, think of an angle and call it x . The complement of this angle is

$$(90 - x)$$

and the supplement of this same angle is

$$(180 - x)$$

Now for Part II read the problem again; this time try to write an equation to describe the Relationship that is stated.

Please return to page $\frac{87}{2}$ and try this question again.

66/1

Did you think that the problem asked for two consecutive "even" integers? If you did, then your answer is correct. But a re-reading of the problem makes it clear that there was no mention of consecutive <u>even</u> integers. It did ask for "consecutive integers" which is a different matter.

Start by letting

x = the first integer

and $x + 1$ = the consecutive integer.

Now read the problem again and write an equation that shows the relationship.

Please return to page 51/2 and try this question again.

66/2

Read the problem carefully. You have to find the angle that equals three times its supplement. The ratio of the two angles is 3:1 , and we want the larger of the two angles.

Please return to page 71/2 and try this question again.

We do not agree. Consider for example, the sum of

(180 - x) and -180

This sum is -x and it should be 180 for the two angles to be supplementary.

There is a second error in your choice, too.

Please return to page 79/2 and try this question again.

This is typical of one of the more complicated problems that you will meet. Don't get panicky. Take the necessary steps, one at a time.

I: THE VARIABLES:

You read that the first angle is three times as large as the second. That gives us the clue to

"Let x = the second angle."

Thus, the first angle can be represented by

3x

O. K. so far? "The Third angle is 12 more than the sum of the first two." Well, don't we have the algebraic expressions for the first two? Find the sum and then add 12 to it. Won't this be the third angle? At this point we are ready for

II: THE RELATIONSHIP:

Well, since the three angles are angles of a triangle, we know that their sum is 180° . There it is!

Of course, you must then solve the equation and finish up by checking the values. Now let's see you do it.

Please return to page 78/2 and attack this problem.

Very good. You made the correct choice.

I: THE VARIABLES:

We start by representing the three consecutive odd integers.

Let

x = the first odd integer,

$x + 2$ = the second,

$x + 4$ = the third.

II: THE RELATIONSHIP:

Now, we are told that three times the sum of the first two exceeds twice the third by 50 . This means that

$$3 (x + x + 2)$$

is 50 more than

$$2 (x + 4)$$

We have the equation,

$3 (x + x + 2) = 2(x + 4) + 50$ [C ^ A

$3 (2x + 2) = 2(x + 4) + 50$ [D

$6x + 6 = 2x + 8 + 50$ [C ^ A

$6x + 6 = 2x + 58$ ∝ - 2x

$4x + 6 = 58$ ∝ - 6

$4x = 52$ ∝ ÷ 4

$x = 13$

Therefore, the numbers are:

$x = 13$

$x + 2 = 15$

$x + 4 = 17$

III: THE CHECK:

(1) The problem stated that they were "consecutive odd integers" ...they are.

(2) "Three times the sum of the first two exceeds twice the third by 50 ."

$3 (13 + 15) \stackrel{?}{=} 2 (17) + 50$

$3 (28) \stackrel{?}{=} 34 + 50$

$84 = 84$ ✓

Please go on to page 69/1 and do question 11.

Question II

Apply your knowledge and find the complements of the angles:

I. 77° II. -110° III. 3d degrees

Select the letter next to the correct answers.

(A) I. 23° II. 80° III. (90 + 3d) degrees

(B) I. 3° II. 200° III. (90 - 3d) degrees

(C) I. 3° II. +20° III. (90 - 3d) degrees

(D) I. 3° II. 70° III. (3d - 90) degrees

This equation will actually lead to the solution. However, you missed a small detail. You were instructed to let x be the angle that is "10° more than its complement." Actually, an examination of your equation shows that you let x be the smaller angle. Of course, if you find the smaller angle, you can then find the larger angle. But, isn't it true, that you did not follow instructions?

Please return to page $\frac{82}{2}$ and try this problem again.

70
1

Very good. You made the correct choice. Note that we are dealing with consecutive even integers.

I: THE VARIABLES

Let

x = the first integer

$x + 2$ = the second integer

$x + 4$ = the third integer

II: THE RELATIONSHIP:

The sum of these integers is 102 . We, therefore, have the equation,

$$x + x + 2 + x + 4 = 102 \quad [\ C \wedge A$$

$$3x + 6 = 102 \quad \prec - \ 6$$

$$3x = 96 \quad \prec \div \ 3$$

$$x = 32$$

$$x + 2 = 34$$

$$x + 4 = 36$$

The largest of these integers, 36 , is exactly divisible by 12 .

III. THE CHECK:

(A) The numbers are consecutive even integers.

32 , 34 , and 36 are consecutive even integers.

(B) Their sum is 102 .

$32 + 34 + 36 = 102$ ✓

Please go on to question 9 below.

70
2

Question 9

Apply your knowledge and solve the following problem:

"Find three consecutive odd integers whose sum is 117 ."

Select the letter next to the correct answer.

(A) The largest integer is divisible by 3 .

(B) The numbers are 39 , 40 , 41 .

(C) The sum of the smallest and largest integers is 78 .

(D) There are no such integers.

Very good. You made the correct choice.

A complete rotation generates an angle of 360 degrees. Now $\frac{2}{3}$ of 360 is 240 . However, a clockwise rotation generates a negative angle. Hence, $\frac{2}{3}$ of a complete clockwise rotation equals -240 degrees. This value is assigned to x .

On the other hand, $\frac{3}{4}$ of a counterclockwise rotation equals +270 degrees, and this value is assigned to y .

Therefore,

$$(x + y) = -240^\circ + 270^\circ$$
$$= 30^\circ$$

Please go on to question 15 below.

Question 15

Apply your knowledge and solve the following problem:

"Find the angle that equals three times its supplement."

Select the letter next to the correct answer.

(A) 135°

(B) 45°

(C) The angle is greater than 45° but less than 90° .

(D) The angle is greater than 90° but less than 120° .

$\frac{72}{1}$

Very good. You made the correct choice.

I. THE VARIABLES:

We start by letting

x = the first consecutive integer,

$x + 1$ = the second consecutive integer.

II: THE RELATIONSHIP:

Re-reading the problem we note that the sum of the two consecutive integers is 37 . We, therefore, have the equation,

$$\begin{aligned} x + x + 1 &= 37 \quad [\, C \wedge A \\ 2x + 1 &= 37 \quad \propto - \ 1 \\ 2x &= 36 \quad \propto \div \ 2 \\ x &= 18 \\ x + 1 &= 19 \end{aligned}$$

III: THE CHECK:

(a) The integers are consecutive; 18 and 19 are such.

(b) And their sum is 37 .

$18 + 19 = 37$ ✓

Please go on to question 7 below.

$\frac{72}{2}$

Question 7

Apply your knowledge and solve the following problem:

"Find three consecutive integers whose sum is 84 ."

Select the letter next to the correct answer.

(A) The sum of the smallest and largest integer is 54 .

(B) The largest integer is 30 .

(C) The smallest integer is divisible by 9 .

(D) The middle integer is divisible by 8 .

Very good. You made the correct choice.

In order for two angles to be called supplementary, the sum of their degree measure must be 180 . Thus,

	Given Angle:	Your Choice:	The Sum:
I:	105°	75°	180°
II.	-80°	260°	180°
III.	(180 - x)°	x°	180°

Please go on to question 13 below.

Question 13

Apply your knowledge and find the number of degrees in the angle made by the hands of a clock at 12:30 .

Select the letter next to the correct answer.

(A) 180°

(B) 0°

(C) 175°

(D) 165°

$\frac{74}{1}$

We do not agree. If two angles are complementary, their sum is $90°$.

Now,

$$3d° + (3d - 90)° = (6d - 90)°$$

That isn't what you meant, is it?

Please return to page $\frac{69}{1}$ and try this question again.

$\frac{74}{2}$

You did not read the question carefully. The complement of 135 degrees is a negative angle while the supplement of 135 degrees is a positive angle. A negative angle cannot equal $\frac{1}{3}$ of a positive angle.

Please return to page $\frac{87}{2}$ and try this question again.

$\frac{75}{1}$

This choice indicates that you did some very fuzzy thinking. Yes, it's true that "complementary angles" do have a relation with $90°$.

$$(90 - x)°$$

would be the complement of x, but what would

$$(90 + x)°$$

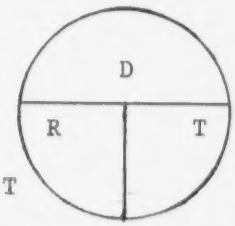
mean? On the other hand, if it were true that those two expressions were the two complementary angles, then their difference would be $10°$. So suppose you clear up that matter of the second angle.

Please return to page $\frac{82}{2}$ and try this problem again.

$\frac{75}{2}$

We do not agree. Recall that the basic relationship is

DISTANCE = RATE x TIME



$R\ T = D \div T$ $\qquad$ $R\ T = D \div R$

$R = \frac{D}{T}$ $\qquad$ $T = \frac{D}{R}$

If you put your finger on the R on the diagram, you find $\frac{D}{T}$

If you put your finger on the T on the diagram, you find $\frac{D}{R}$

If you put your finger on the D on the diagram, you find R T

This device becomes a "memory aid" while you are learning the rules. However, it is better to use the techniques of solving equations on the basic relationship

$$D = R\ T$$

and derive the other two rules as you need them.

Please return to page $\frac{89}{2}$ and try this question again.

$\frac{76}{1}$

We do not agree.

Start by letting

x = the required angle, and

$(180 - x)$ equal its supplement.

Now, read the problem again and note how these two angles are related. This relationship can be expressed as an equation. Solve the equation and check your answers with the words of the problem.

Please return to page $\frac{71}{2}$ and try this question again.

$\frac{76}{2}$

We do not agree. How should the angles of this triangle be represented? Note that we are dealing with consecutive even integers. Thus, if we denote the number of degrees in the smallest angle by x, the next larger angle is denoted by

$$x + 2$$

After you have represented the three angles of the triangle algebraically, you must then relate them to another restricting fact. The sum of the angles of the triangle is a fixed idea. Use that.

Please return to page $\frac{85}{1}$ and try this question again.

Very good. You made the correct choice.

I. THE VARIABLES:

Since we are dealing with odd consecutive integers, we let

x = the first odd integer

$x + 2$ = the second odd integer

$x + 4$ = the third odd integer

Some students feel that the addition of 2 results in obtaining even integers. Somehow, one associates 2 with even integers. However, one must keep in mind that we are starting with an odd integer, and adding 2 to an odd integer yields the next odd integer.

II: THE RELATIONSHIP:

Returning to our problem, we have the equation, since "the sum = 117."

$$x + x + 2 + x + 4 = 117 \qquad [\ C \wedge A$$

$$3x + 6 = 117 \qquad \prec - \ 6$$

$$3x = 111 \qquad \prec \div \ 3$$

$$x = 37$$

$$x + 2 = 39$$

$$x + 4 = 41$$

III: THE CHECK:

(1) The numbers are consecutive odd integers

37 , 39 , and 41 are consecutive odd integers

(2) Their sum is 117 :

$37 + 39 + 41 = 117$ ✓

Please go on to question 10 below.

Question 10

Apply your knowledge and solve the following problem:

"Find three consecutive odd integers such that three times the sum of the first two exceeds twice the third by 50 ."

Select the letter next to the correct answer.

(A) Two of the integers are 17 and 19 .

(B) The sum of the integers is 45 .

(C) The largest integer is 19 .

(D) The smallest integer is 15 .

$\frac{78}{1}$

Very good. You made the correct choice.

I. THE VARIABLES:

We start by letting,

x = the required angle

$90 - x$ = the complement of this angle.

$180 - x$ = the supplement of this angle.

II. THE RELATIONSHIPS:

Now, we are told that the complement is equal to $\frac{1}{3}$ the supplement. We, therefore, can write the equation,

$$90 - x = (\tfrac{1}{3})(180 - x) \quad \times 3$$

$$3(90 - x) = 180 - x \quad [D$$

$$270 - 3x = 180 - x \quad \times +3x$$

$$270 = 180 + 2x \quad \times -180$$

$$90 = 2x \quad \times \div 2$$

$$45 = x$$

Conclusion:

$$x = 45$$

$$90 - x = 45$$

$$180 - x = 135$$

III: THE CHECK:

The complement is $\frac{1}{3}$ of the supplement

$$45 \stackrel{?}{=} \tfrac{1}{3}(135)$$

$$45 = 45 \quad \checkmark$$

Please go on to question 18 below.

$\frac{78}{2}$

Question 18

Apply your knowledge and solve the following problem:

"One angle of a triangle is three times as large as the second, and the third angle is $12°$ more than the sum of the other two.

Find the angles of the triangle:

(A) 16 (B) 20

(C) 21 (D) 28

Very good; your choice is correct.

Two angles are said to be complementary when the sum of their degree measures is $90°$. (Note: there is no restriction in that definition that the angles must be less than $90°$ each, although this is the case in applications of this relationship in trigonometry.)

Given Angle:	Your Choice	Their Sum
$77°$	$13°$	$90°$
$-110°$	$200°$	$90°$
$3d°$	$(90 - 3d)°$	$90°$

Please go on to question 12 below.

Question 12

Apply your knowledge and find the supplement of the angles:

I. $105°$ II. $-80°$ III. $(180 - x)$ degrees

Select the letter next to the correct answers.

(A)	I. $85°$	II. $-100°$	III. $0°$
(B)	I. $85°$	II. $100°$	III. $180°$
(C)	I. $75°$	II. $260°$	III. $x°$
(D)	I. $75°$	II. $100°$	III. $-180°$

$\frac{80}{1}$

Very good. You made the correct choice.

Let us make a picture of a clock.

Firs, we note that the twelve numbers divide the clock into twelve equal arcs and each arc, therefore, has

$$\frac{360^\circ}{12} = 30^\circ$$

Now, at 12:30 the minute hand is exactly at 6 but the hour hand is midway between 12 and 1 . That is, the hour hand has moved 15° away from 12 . Thus, the angle formed is

$$180^\circ - 15^\circ = 165^\circ$$

Please go on to question 14 below.

$\frac{80}{2}$

Question 14

Apply your knowledge and find the number of degrees in (x + y) if the measures of x and of y are as follows:

$x = \frac{2}{3}$ of a complete rotation clockwise

$y = \frac{3}{4}$ of a complete rotation counterclockwise.

Select the letter next to the correct answer.

(A) 60° (C) 30°

(B) -30° (D) -60°

Yes, your choice is correct.

Thus, the rate of Mr. Q is

$$\text{RATE} = \frac{\text{DISTANCE}}{\text{TIME}}$$

$$\frac{160}{3\frac{3}{4}} = \frac{160}{\frac{15}{4}} = 160 \times \frac{4}{15} = \frac{640}{15} = 42\frac{2}{3}$$

and the rate of Mr. R is

$$\frac{175}{4\frac{1}{2}} = \frac{175}{\frac{9}{2}} = 175 \times \frac{2}{9} = \frac{350}{9} = 38\frac{8}{9}$$

Hence, Mr. R traveled at less than 40 miles per hour.

Please go on to question 2 below.

Question 2

Apply your knowledge and select the letter next to the correct answer to the following question:

"Mr. X traveled a distance of 880 miles at an average of 60 miles per hour, while Mr. Y traveled a distance of 1280 miles at an everage of 75 miles per hour: what can be said about the time of their trips?"

(A) Mr. X's trip took longer.

(B) Mr. X's trip took over 15 hours.

(C) Mr. Y's trip took over 18 hours.

(D) Both trips took a total of less than 32 hours.

82
1

Very good. You made the correct choice.

I. THE VARIABLES:

We start by letting

x = the number of degrees in the required angle

$180 - x$ = the number of degrees in the supplement.

II. THE RELATIONSHIP:

The problem states that the angle is three times its supplement.

We, therefore, have the equation,

$$x = 3(180 - x) \quad [D$$
$$x = 540 - 3x \quad \measuredangle + 3x$$
$$4x = 540 \quad \measuredangle \div 4$$
$$x = 135$$

III. THE CHECK:

The supplement of 135 is 45, and 135 equals 3 (45), which agrees with the condition: "the angle is three times its supplement."

Please proceed to question 16 below.

82
2

Question 16

Apply the proper principles and solve the following problem:

"Find the angle x that equals 10 more than its complement."

Select the letter next to the equation describing the relationship.

(A) $x = (90 - x) + 10$

(B) $(x) + (90 - x) = 90$

(C) $(x) + (10 + x) = 90$

(D) $(90 - x) - (90 + x) = 10$

The first job is to read the question carefully. Note that the trains start from the same point and travel in opposite directions. Make a little sketch to help see what is happening: thus,

You must determine the relationship between the two distances.

Please continue.

Please return to page $\frac{92}{2}$ and try this question again.

If a plane travels 2000 miles in four hours, how far does it go in one hour? Yes,

$$\frac{2000}{4} = 500 \text{ miles}$$

How did we get this answer? We divided the distance by the time. Similarly, if a plane travels 2000 miles in h hours, how far will it travel in one hour?

Place the information in a chart form.

Please return to page $\frac{94}{2}$ and try this question again.

Very good. You made the correct choice in a fairly complicated problem. Let's review the method. We start as follows:

I: THE VARIABLES:

Let x = the number of degrees in the second angle,

let $3x$ = the number of degrees in the first angle,

then $4x$ = the number of degrees in both these angles.

Now, the third angle is twelve degrees more than this sum. Hence,

$4x + 12$ = the number of degrees in the third angle.

II: THE RELATIONSHIP:

Using the fact that the number of degrees in all the angles of a triangle is 180 we have the equation:

1st Angle	+	2nd Angle	+	3rd Angle	=	180°	
$3x$	+	x	+	$(4x + 12)$	=	180	[C ^ A
		$8x$	+	12	=	180	∝- 12
		$8x$			=	168	∝÷ 8
		x			=	21	

Conclusion:

1st angle	$3x$	=	63°
2nd angle	x	=	21°
3rd angle	$4x + 12$	=	96°

III: THE CHECK:

(1) The first angle is three times the second

$63 = 3(21)$ ✓

(2) The third angle is 12 more than the sum of the first two

$96 \stackrel{?}{=} 12 + (63 + 21)$

$96 \stackrel{?}{=} 12 + 84$

$96 = 96$ ✓

(3) The sum of the three angles of a triangle = 180°

$63 + 21 + 96 \stackrel{?}{=} 180°$

$180 = 180$ ✓

Please go on to the next question on page 85/1 .

Question 19

Apply your knowledge and solve the following problem:

The angles of a triangle are consecutive even integers: find the number of degrees in each angle.

Select the letter next to the correct answer.

(A) The smallest angle is $56°$.

(B) The largest angle is $64°$.

(C) The smallest angle plus the largest is $120°$.

(D) The measures of the angles cannot be integers.

We do not agree. It is important to know how to convert from one set of units to another. If a ski lift runs 22 feet in one second, how far does it go in one minute (60 seconds) ?

Yes, 60 times as far.

How long will it take to cover a mile is the next consideration.

Please return to page $\frac{88}{2}$ and try this question again.

$\frac{86}{1}$

We do not agree.

TIME = DISTANCE DIVIDED BY RATE

Thus,

Mr. X's time = $\frac{880}{60}$

and Mr. Y's time = $\frac{1280}{75}$

You will find it helpful in division problems of this type to "reduce the fractions" first. This means that the numerator and denominator must be factored; then the common factor can be removed.

For example:

$$\frac{P}{Q} = \frac{a \cdot b}{c \cdot b} = \frac{a}{c} \times \frac{b}{b} = \frac{a}{c} \times 1 = \frac{a}{c}$$

Please go over your division of the above quotients.

Please return to page $\frac{81}{2}$ and try this question again.

$\frac{86}{2}$

This is an example of a two stage problem. You use the information given to find the rate per hour from the first condition. Then this derived information is used in the second part to arrive at the time. Place the given facts in a chart form and the relationships will become obvious.

	Rate	Time	Distance
I:	x	$1\frac{2}{3}$	960
II:	x	y	2016

Now return to page $\frac{99}{2}$ and try this problem again.

Very good. You made the correct choice.

I. THE VARIABLES:

If we

let x = the required angle,

then $90 - x$ = its complement.

II. THE RELATIONSHIP:

Now, the angle is 10 degrees more than its complement. Hence,

$$x = (90 - x) + 10 \qquad [\ C \wedge A$$

We can continue with the solution to verify our equation:

$$x = 100 - x \qquad \text{add } +x$$

$$2x = 100 \qquad \text{divide by } 2$$

$$x = 50$$

and $90 - x = 40$

III. THE CHECK:

(1) The angles are complementary,

$40° + 50° = 90°$ ✓

(2) One angle is $10°$ more than the other,

$50°$ is $10°$ more than $40°$ ✓

Please go on to question 17 below.

Question 17

Apply your knowledge and solve the following problem:

"Find the angle x whose complement equals $\frac{1}{3}$ of its supplement."

Select the letter next to the correct answer.

(A) $50°$

(B) $135°$

(C) Greater than $120°$ but less than $135°$.

(D) Great than $30°$ but less than $50°$.

88
1

Yes, your choice is correct.

I: THE VARIABLES:

The given information can be arranged in chart form:

	Rate	Time	Distance
TRAIN 1	60	8	60 • 8
TRAIN 2	40	8	40 • 8

II: THE RELATIONSHIP:

Since the trains start from the same place and move in opposite directions, the distance apart will be the sum of the distances traveled by the trains. The sum of 480 and 320 is 800 .

Please go on to question 4 below.

88
2

Question 4

Apply your knowledge and select the letter next to the correct answer to the following question:

A ski lift runs at the rate of 22 feet per second; at that rate, how minutes would it take to cover a mile? (1 mile = 5,280 feet)

(A) 24 minutes

(B) 6 minutes

(C) 4 minutes

(D) 11 minutes

2

VOLUME 6 SEGMENT 3 begins here:
Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	2 8	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 6	(Volume Number)
	66	and	68	0 3	(Segment Number)

Your READING ASSIGNMENT for this Segment is pg: 178 - 180 .

SUPPLEMENTARY NOTE:

In this segment, we are going to deal with a most interesting type of problem called uniform motion. How often have you heard expressions such as: there is a great distance between the Earth and the Moon; the plane traveled at great speed; the trip to the country took a long time.

Notice the three items comprising motion; distance, rate, and time. Reading your text assignment, you have found out how these three items are related; namely,

DISTANCE = UNIFORM RATE x TIME

In all the problems that you will be asked to solve in this segment, you will use the above relation. Furthermore, in order to successfully cope with these probelms, you must see what is happening. Read the problem and make a sketch that pictures the situation. One further bit of advice:

DO NOT MAKE UP YOUR MIND THAT YOU CAN'T DO THE PROBLEM.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Apply your knowledge and select the letter next to the correct answer to the following question: " Mr. Q covered 160 miles in $3\frac{3}{4}$ hours, while Mr. R covered 175 miles in $4\frac{1}{2}$ hours; what can be said about their average speeds? "

(A) Mr. R traveled faster.

(B) Both men traveled at the same rate.

(C) Mr. Q traveled at more than 45 miles per hour.

(D) Mr. R traveled at less than 40 miles per hour.

Very good. You made the correct choice.

I: THE VARIABLES:

Since we are dealing with consecutive even integers, we let

x = the number of degrees in the first angle,

$x + 2$ = the number of degrees in the second angle,

$x + 4$ = the number of degrees in the third angle.

II: THE RELATIONSHIP:

Using the fact that the number of degrees in the sum of the three angles of a triangle is 180 we obtain the equation,

$$(x) + (x + 2) + (x + 4) = 180 \quad [C \wedge A$$

$$3x + 6 = 180 \quad \prec -6$$

$$3x = 174 \quad \prec \div 3$$

$$x = 58$$

Conclusion:

$$x = 58$$

$$x + 2 = 60$$

$$x + 4 = 62$$

III: THE CHECK:

(a) The angles are consecutive even integers.

58 , 60 , and 62 are such.

(b) The sum of the angles of a triangle is 180° .

$$58 + 60 + 62 \stackrel{?}{=} 180$$

$$180 = 180 \checkmark$$

Please go on to the next page.

You have now finished this segment. Hand in your PUNCH CARD. You should have entered in your NOTEBOOK the following definitions and formulas:

(1) An integer is another name for any whole number; positive, negative, or zero.

(2) Consecutive means following in order. Consecutive integers are obtained by counting by ones.

(3) Two angles are complementary if the sum of their degree measures is 90 .

(4) Two angles are supplementary if the sum of their degree measures is 180 .

(5) Counterclockwise rotation yields a positive directed angle; clockwise rotation yields a negative directed angle.

(6) The sum of the measures of the angles of any triangle is 180 .

The General Procedure for solving a VERBAL PROBLEM:

(1) Read the problem carefully more than once.

(2) Draw a diagram and label it (if it is possible).

(3) Part I: THE VARIABLES:
   (a) Select the independent variable and call it x or some other appropriate letter.
   (b) Express the other variables in terms of x .

(4) Part II: THE RELATIONSHIP:
   (a) From the diagram or from the nature of the facts in the problem, state a Relationship that exists among the variables.
   (b) Substitute the Variables in the Relationship.

(5) Solve the equation using indicated operations

(6) Part III: THE CHECK:
   (a) Check the values of the variables themselves.
   (b) Check the values of the variables in the RELATIONSHIP.

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT: problems 5 , 6 , 7 and 8 .

92/1

Very good. You made the correct choice.

We use the fact that

$$\text{TIME} = \frac{\text{DISTANCE}}{\text{RATE}}$$

	Mr. X's time		Mr. Y's time	
	$\frac{880}{60}$		$\frac{1280}{75}$	
Total time:	$14\frac{2}{3}$	+	$17\frac{1}{15}$	=
	$14\frac{10}{15}$	+	$17\frac{1}{15}$	$= 31\frac{11}{15}$ hours

This sum is less than 32 hours.

Please go on to question 3 below.

92/2

Question 3

Apply your knowledge and select the letter next to the correct answer to the following question:

" Two trains start from the same place at the same time and travel in opposite directions. The first train goes at a uniform rate 60 miles an hour, and the second train goes at a uniform rate 40 miles an hour; how far apart will they be in eight hours ? "

(A) 700 miles (C) 160 miles

(B) 800 miles (D) 100 miles per hour

This is not an easy question; it is, however, a very important one. It seems that the correct answer should be the average of the two speeds; that is,

$$\frac{10 + 20}{2} = 15 \text{ miles per hour}$$

This is not the case. We must keep in mind that the average speed of <u>any</u> trip is the quotient of the <u>total distance</u> and the <u>total time</u>.

Return to page 114/2 and try this question again.

Let us suppose for a moment that your choice is correct.

	R	T	D	
I:	30	4	(120)	The car going at 30 mph for four hours will cover a distance of 30 x 4 = 120 miles.
II:	35	4	(140)	The car going at 35 mph for four hours will cover 35 x 4 = 140 miles.
TOTAL			(260)	Now, 120 + 140 = 260

The total distance they were apart at the start was 400 miles. Therefore, if they have traveled a total of 260 miles, they remain 140 miles apart.

Note:

(1) Although the units must be kept in mind, and sometimes are included in the heading for the particular problem, the actual numbers that are inserted in the chart are pure numbers without the units indicated.

(2) To distinguish the numbers that are given from the numbers that are derived, a pair of parentheses are used around the derived number.

Please return to page 111/1 and try this question again.

$\frac{94}{1}$

Very good. You made the correct choice.

If a plane can cover 960 miles in $1\frac{2}{3}$ hours we can exhibit this information in a chart form:

	Rate	Time	Distance	
I:	x	$1\frac{2}{3}$	960	$(1\frac{2}{3})x = 960$

$$\frac{5}{3}x = 960 \quad \times 3$$

$$5x = 2880 \quad \div 5$$

$$x = 576$$

We can use this derived information to solve the second part of the problem.

	Rate	Time	Distance	
II:	(576)	y	2016	$576y = 2016 \quad \div 576$

$$y = 3\frac{1}{2}$$

Note:

(1) Although the " units " must be kept in mind, and sometimes are included in the heading for the particular problem, the numbers that are inserted in the chart are pure numbers without the units indicated.

(2) To distinguish the numbers that are given from the numbers that are derived, a pair of parentheses are used around the numbers that are derived. This is another use of parentheses.

Please go on to question 6 below.

$\frac{94}{2}$

Question 6

Apply your knowledge and select the letter which correctly completes the statement: If a plane travels 2000 miles in h hours, the time it will take the plane to travel 5000 miles expressed in terms of h is _____.

(A) 5000 h hours

(B) $(\frac{5}{2})$h hours

(C) $(\frac{2}{5})$ h hours

(D) 2000 h hours

This is not a good answer. A simple check will disprove your choice. Let's place the given facts in a chart:

	R	T	D
F	60	$8\frac{1}{3}$	(500)
G	(48)	$8\frac{1}{3}$	(400)

The total distance should equal 975 miles, but as you can see; it doesn't. Perhaps you made a mistake in arithmetic. 8 hours and 20 minutes should be converted to 8 and $\frac{1}{3}$ hours. After finding the distance that the Train F covered, the remaining distance must be covered by the other trian.

Note:
(1) Although the " units " must be kept in mind, and sometimes are included in the heading for the particular problem, the actual numbers in the chart are pure numbers without the units indicated.

(2) To distinguish the numbers that are given from the numbers that are derived, a pair of parentheses are used around the derived numbers.

Return to page 117/1 and try this problem again.

Please be careful with the units of time. Two hours is 120 minutes, and in this time Achilles will be many yards ahead of the tortoise.

Return to page 124/2 and try this question again.

$\frac{96}{1}$

If instead of the letters m and h , we used numbers, you probably would have no trouble with this question. To help your thinking, replace the letters with numbers.

Please return to page $\frac{100}{2}$ and try this question again.

$\frac{96}{2}$

Do we know how long Mr. X traveled at each rate? No. However, we do know that the entire trip took $5\frac{1}{2}$ hours. Thus, if we

let x = the time Mr. X traveled at 50 mph,

then $(5\frac{1}{2} - x)$ = the time he traveled at 40 mph.

Note that it is more convenient to replace $5\frac{1}{2}$ by $\frac{11}{2}$.

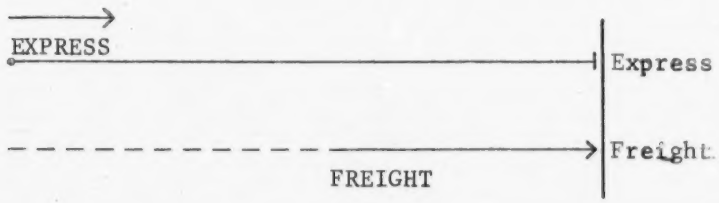
Please continue.

Please return to page $\frac{118}{2}$ and try this question again.

We do not agree. You were asked for the distance apart, but you gave the rate of separation of the two trains.

Return to page 92/2 and try this question again.

Let us make a sketch which tells the story of the given situation. Thus,



The freight train has a three hour head start. The express train traveling at a much faster rate will eventually overtake the freight train at a certian place on the line. When this happens, both trains will momentarily be at the same place and will each have traveled the same distance. Now,

let x = the time it will take the express to catch the freight train.

Place all the given facts into a chart and continue.

Return to page 119/2 and try this question again.

98
1

Very good. You made the correct choice. The important fact to note is that the average speed of any trip whatsoever is the quotient of the total distance and the total time. We don't know the distance, but it's the same up and down the hill. Let's call it d . Let the time up be " p " and the time down be " q " .

		Rate	Time	Distance	time up:	time down:
I:	(up)	10	p	d	$10p = d$ $\lessdot \div 10$	$20q = d$ $\lessdot \div 20$
II:	(down)	20	q	d	$p = \frac{d}{10}$	$q = \frac{d}{20}$

Now, the average rate will be called r .

III: TOTAL $r\ (p + q) = 2d$

$r\ (p + q) = 2d$

$r\ \left(\frac{3d}{20}\right) = 2d \lessdot \cdot 20$

$r \cdot 3d = 40d \lessdot \div 3d$

$r = \frac{40d}{3d}$

$r = \frac{40}{3}$

$r = 13\frac{1}{3}$

Note: total time:

$p + q = \frac{d}{10} + \frac{d}{20}$

$= \frac{2d}{20} + \frac{d}{20}$

$= \frac{3d}{20}$

Now proceed to the next question below.

98
2

Question 9

Apply your knowledge and solve the following problem:

" Two trains start at the same time on parallel tracks from two towns that are 1232 miles apart. One train travels at 64 miles per hour and the other train at 48 miles per hour. In how many hours will they meet? "

Select the letter next to the correct answer.

(A) 12 hours

(B) 14 hours

(C) In more than 12 but less than 14 hours.

(D) In less than 12 but more than 10 hours.

Yes, your choice is correct.

If a ski lift runs at the rate of 22 feet per second, then in one minute it runs 60 times as far, since there are 60 seconds in one minute.

22 feet in 1 second x 60

1320 feet in 1 minute

To find the number of minutes it takes for the lift to cover a mile, we must find how many times 1320 feet occur in a mile; that is, we must divide one mile by 1320 feet.

i.e. $\frac{5280 \text{ feet}}{1320}$ or 4

This figure then multiplies the relationship above:

1320 feet in 1 minute x 4

5280 feet in 4 minutes

or 1 mile in 4 minutes

Please go on to question 5 below.

Question 5

Apply your knowledge and select the letter next to the correct answer to the following question:

" If a plane can cover 960 miles in $1\frac{2}{3}$ hours, how long will a 2016 mile trip take? "

(A) $5\frac{1}{3}$ hours (C) $3\frac{1}{2}$ hours

(B) $3\frac{2}{3}$ hours (D) 35 hours

$\frac{100}{1}$

Very good. You made the correct choice.
If a plane travels 2000 miles in h hours, in one hour the plane will travel:

	Rate	x	Time	=	Distance	
I:	r		h		2000	
	r	x	h	=	2000	$\propto \div$ h
	r	=	$\frac{2000}{h}$ miles			

This is the rate of the plane.

II:

If we want to find the time it will take the plane to make a 5000 mile trip, we resort to the chart again, and we use that rate from I .

Rate	x	Time	=	Distance
$(\frac{2000}{h})$		t		5000

$$\frac{2000}{h}\ (\ t\ ) = 5000 \propto h$$

$$2000\ t = 5000\ h \propto \div 2000$$

$$t = \frac{5}{2}\ h$$

Note:

(1) Although the " units " must be kept in mind, and sometimes are included in the heading for the particular problem, the actual numbers that are inserted in the chart are pure numbers without the units indicated.

(2) To distinguish the numbers that are given from the numbers that are derived, a pair of parentheses are used around the numbers that are derived and placed in the chart.

Please go on to question 7 below.

$\frac{100}{2}$

Question 7

Apply your knowledge and select the letter next to the correct answer to the following question: If a train travels m miles in h hours, how far does it go in five hours?

(A) $\frac{5m}{h}$ miles (C) 5mh miles

(B) $\frac{5h}{m}$ miles (D) None of these.

This choice is not correct. Let's place these values in a "Check Chart" and demonstrate the error.

	R	T	D
F	60	$8\frac{1}{3}$	(500)
G	(60)	$8\frac{1}{3}$	(500)

The problem indicates that the trains met after one covered one part of the distance and the other covered the remaining part. Therefore, we can conclude that the total distance covered by the two trains equals the distance given between New York and Chicago, 975 miles. But it doesn't look that way in your check, does it?

NOTE:

(1) Although the "units" must be kept in mind, and sometimes are included in the heading for the particular problem, the actual numbers in the chart are pure numbers without the units indicated.

(2) To distinguish the numbers that are given from the numbers that are derived, a pair of parentheses are used around the numbers that are derived and placed in the chart.

Please return to page $\frac{117}{1}$ and try this problem again.

It looks like you had some difficulty setting up the problem. Let's give you a push in the right direction.

The key is in the question, "How fast did each train travel?" Since you were told that one train was twice as fast as the other, we can let the rate of the slow one be x and then the rate of the other can be called $2x$.

Next, consider the fact that the total time was given, but not the individual times. Also the total distance each way was given. Now, there's a clue. Can't you express the individual times in terms of the distance and the rates (which you already labeled)?

Place this information on a chart and take it from there.

Please return to page $\frac{123}{2}$ and continue with the problem.

$\frac{102}{1}$

We do not agree. Your answer does not check.

In five hours the car going at 30 mph will travel 150 miles, and the car going at 35 miles per hour will travel 175 miles. Now,

$$150 + 175 = 325 \text{ miles}$$

and the two cars will be 75 miles apart instead of 10 miles.

	R	T	D
I:	30	5	(150)
II:	35	5	(175)
TOTAL:			(325)
400 - 325		=	75

Please return to page $\frac{111}{1}$ and try this question again.

$\frac{102}{2}$

We do not agree. In $2\frac{1}{3}$ minutes Achilles will travel a distance of

$$500 \times 2\frac{1}{3} = 500 \times \frac{7}{3} = 1166\frac{2}{3} \text{ yards.}$$

In $2\frac{1}{3}$ minutes the tortoise will travel

$$5 \times 2\frac{1}{3} = 5 \times \frac{7}{3} = 11\frac{2}{3}$$

Thus, with a head start of 990 yards, the tortoise will be

$$990 + 11\frac{2}{3} = 1001\frac{2}{3} \text{ yards from the starting point.}$$

Hence, Achilles will have passed the tortoise.

Please return to page $\frac{124}{2}$ and try this question again.

At the rate of 2 miles per hour, a 25 mile hike would take $12\frac{1}{2}$ hours. Then we still have the time it would take to bicycle back. Since the round trip took only three hours, your answer does not check.

Start by letting

x = the time it takes the boy to walk to the lake and

$3 - x$ = the time it takes to return.

Note that if the total time is known, and part of this time is denoted by x, the rest of the time is denoted by the total minus x.

Use the chart form and examine it for a relationship that will lead to an equation.

Please return to page $\frac{115}{2}$ and try this question again.

The word value has a precise meaning. It means the cost per unit times the number of units. Algebraically, we can write

$$N \times R = V$$

where V stands for value, R the cost of one unit, and N stands for the number of units.

Now, if 100 pounds of coffee costs \$65, what is the cost of 1 pound?

What is the value of 80 pounds?

Construct a chart for organization and analysis of this information.

Please return to page $\frac{130}{2}$ and try this question again.

104/1

Your answer is more than three hours. Suppose the express train travels for only three hours; it will cover a distance of 150 miles. Now, the freight train with a three hour head start will travel for six hours and cover

$$6 \times 20 = 120 \text{ miles}$$

Thus, in three hours the express will be 30 miles ahead of the freight train.

Please return to page 119/2 and try this question again.

104/2

We do not agree. Let us talk about the problem for a moment. There are two trains traveling toward each other at different rates of speed. In six hours these trains will cover a combined distrance of 726 miles. We are asked to find the rates of these trains. If we

let x = the rate of the slower train,

how should we denote the rate of the faster train?
What is the distance covered by each of the trains?
Answer these questions, and you will be able to write the correct equation.

Please return to page 113/2 and try this question again.

Please note that the rate equals distance divided by time. The rate of the train is the distance m divided by the time h . Insert this information on the chart form and attack part II .

Please return to page 100/2 and try this question again.

You will be surprised, but careful analysis and careful placement of the given facts in the "chart" will give you enough information to arrive at an exact answer for the average time of Train G .

Please return to page 117/1 and try this problem again.

106/1

Your answer does not check. If Mr. X traveled for $2\frac{1}{2}$ hours at 50 miles per hour, he would cover a distance of

$$(2\frac{1}{2})\ (50) = 125 \text{ miles}$$

There are three hours left for traveling at the rate of 40 mph, and

$$3\ (40) = 120$$

Then Mr. X would cover

$$125 + 120 = 245$$

instead of only 240 miles.

	R	T	D
I	50	$(2\frac{1}{2})$	(125)
II	40	(3)	(120)
TOTAL	-	$5\frac{1}{2}$	(245) x

Please return to page 118/2 and try this question again.

106/2

This problem is a real challenge. Begin by making two separate diagrams; then for each diagram make a separate chart,

letting x = the quantity that you are seeking, the distance.

Note: when you know the rate and have the distance expressed as a variable, it is possible to express the time by the rule:

$$T = \frac{D}{R}$$

Since there is a relationship expressed between the two total times, you should be able to construct an equation. If you have gotten this far, it is highly probable that you will have to clear fractions by multiplying both sides of the equation by 84.

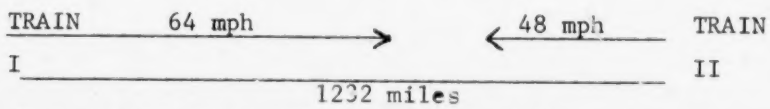
Please return to page 127/1 and tackle this problem.

It is important that you develop a procedure for solving this type of problem.

SKETCH:

Read the problem and make a sketch describing what is happening.

Thus,



Observe that the sketch shows two trains that are 1232 miles apart and are traveling toward each other at rates of 64 mph and 48 mph respectively.

I. THE VARIABLES:

Now, read the problem again. This time you are concerned with what you are asked to find. You are asked to find the time it takes for the two trains to meet.

Let x = that time.

At this point, you set up a chart of the given facts.

II: THE RELATIONSHIP:

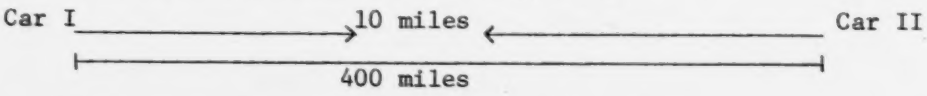
You read the problem once more and this time determine the relationship and write the equation.

	R	x	T	=	D
TRAIN A					
TRAIN B					

Check your answer with the words of the problem.
Set up a second chart with the newly found values.

Please return to page 98/2 and try this question again.

To get you started, let us visualize what is happening.



The two cars pictured above must together cover 390 miles.

Let x = the time it takes the two cars to be ten miles apart.

What distance will car I cover in this time? What distance will car II cover in this time? What can be said about the sum of these distances?

	R	T	D
I	30	x	
II	35	x	
Distance traveled:			390

Please continue.

Return to page 111/1 and try this question again.

108
1

Start by letting

x = the time it will take Achilles to catch the tortoise.

Now, in this time, the tortoise travels 5x yards. Do not forget that the tortoise has a handicap of 990 yards. Please continue with the "Relationship."

	R	T	D
A	500	x	500x
T	5	x	5x

Please return to page 124/2 and try this question again.

108
2

You are not answering the question. You are asked to find the distance that the boy walked, and not how long he walked. After you have found the value of the unknown, say x , you must check to see what this unknown denotes. That is why we insist that you write out in detail what the unknown stands for.

Do not write

let x = time

Write let x = the time it took the boy to hike to the lake.

Please return to page 115/2 and try this question again.

We do not agree. One of the letters does have the correct answer next to it. Please reconsider your choice. Remember, use the chart form to organize your thinking.

Please return to page 100/2 and try this question again.

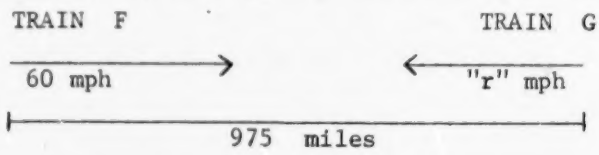
We do not agree. One of the letters does have the correct answer next to it.

Begin the chart:

	R	T	D
F	20	x + 3	
E	50	x	

Good, that choice is correct. Let's review the procedure.

The SKETCH: We start by making a sketch of the situation:



I: THE VARIABLES:

We are looking for the average speed of Train G, let it be called "r".
Since this is a motion problem dealing with average rates of speed, we can use the Rule: D = R T

THE CHART:

	R	T	D
F	60	$8\frac{1}{3}$	
G	r	$8\frac{1}{3}$	

II: THE RELATIONSHIP:

The entire distance is 975 miles. When they meet, one train will have covered part of the distance and the other train, the remaining part.

(1) The distance for F is $60 \times 8\frac{1}{3} =$

$$60 \times \frac{25}{3} = 500 \text{ miles}$$

(2) The distance remaining for G is $975 - 500 = 475$

(3) This can be inserted in the diagram above, and we have the equation:

$$r\left(8\frac{1}{3}\right) = 475$$

$$\frac{25}{3}r = 475 \quad \text{«÷25}$$

Note: this simplifies the arithmetic. We could have multiplied by 3 first, but the numbers would be larger.

$$\frac{r}{3} = 19 \quad \text{«·3}$$

$$r = 57$$

THE CHECK:

	R	T	D
F	60	$8\frac{1}{3}$	(500)
G	(57)	$8\frac{1}{3}$	(475)
Total distance:			(975)

Please proceed to question 11 on page $\frac{111}{1}$.

Question 11

Apply your knowledge and solve the following problem:

"If two cars drive toward each other from points 400 miles apart at rates of 30 mph and 35 mph respectively, after how many hours will they be 10 miles apart?"

Select the letter next to the correct answer.

(A) 4 hours

(B) 5 hours

(C) More than 5 , but less than 6 hours

(D) More than $5\frac{1}{2}$, but less than $6\frac{1}{4}$ hours.

We do not agree. One of the letters does have the correct answer next to it.

Let x = the time he traveled at 50 mph , then the remaining time would be

$$\left(5\frac{1}{2} - x\right) \text{ or } \left(\frac{11}{2} - x\right)$$

Insert these in the CHART and determine the RELATIONSHIP for the equation.

Please return to page 118/2 and try this question again.

112
1

Yes, your choice is correct.

You were given that 100 pounds of coffee has a value of $65.

This information can be placed in chart form for analysis:

	N	R	V
(A)	100	$\frac{65}{100}$	65
(B)	85	(.65)	85 (.65)

$R = \frac{V}{N}$

To obtain the cost per pound (the rate), the Value is divided by the Number.

The Value is found by multiplying the new Number of pounds by the unit cost just found.

Value = $52.00

NOTE:

(1) Although the units must be kept in mind and sometimes are included in the heading for the particular problem, the actual numbers inserted in the chart are pure numbers without the units indicated.

(2) To distinguish the numbers that are given from the numbers that are derived a pair of parentheses are used around the derived numbers.

Please go on to question 2 below.

112
2

Question 2

Apply your knowledge and select the letter next to the correct answer to the following question:

"One bin contains x pounds of candy worth 80¢ a pound. A second bin contains candy worth 95¢ a pound. If both bins contain 150 pounds in all, what is the value (in cents) of the more expensive candy?"

(A) 95 (150)

(B) 95 (x - 150)

(C) 95 (150 - x)

(D) 95 (80 - x)

Very good. You made the correct choice.

I. THE VARIABLES:

Let us start by making a sketch that describes the situation:

Let x = the time it takes Achilles to catch the tortoise
Placing all the facts into a chart, we get

Rule:	R	x	T	=	D
Archilles	500		x		500x
Tortoise	5		x		5x

II: THE RELATIONSHIP:

The distance that Achilles covers must equal the distance covered by the tortoise plus the handicap. Thus, we have the equation,

$$500x = 5x + 990 \quad \text{<- } 5x$$

$$495x = 990 \quad \text{<÷ } 495$$

$$x = \frac{990}{495} = 2$$

III THE CHECK:

In two minutes Achilles travels 500 x 2 = 1000 yards;
in two minutes the tortoise travels 5 x 2 = 10 yards:
and 990 = 10 = 1,000 yards

Please go on to question 14 below.

Question 14

Apply your knowledge and solve the following problem:

"Two trains start from two towns 726 miles apart and meet in six hours. If one train goes nine miles per hour faster than the other, find their rates."

Select the letter next to the correct answer.

(A) The faster train goes at 60 mph.
(B) The slower train goes at 60 mph.
(C) The faster train travels at a speed greater than 64 mph.
(D) The combined speeds of both trains is 120 mph.

114
1

Very good. You made the correct choice.

If a train travels m miles in h hours, then in one hour, the train will travel $\frac{m}{h}$ miles. Thus, the rate of the train is $\frac{m}{h}$.

To find the distance, we multiply the rate by the time.

Hence, the distance the train travels is

$$5\left(\frac{m}{h}\right) \text{ miles}$$

	Rate	Time	Distance		
I:	r	h	m	$rh = m$	$\propto \div h$
II:	$\left(\frac{m}{h}\right)$	5	d	$r = \frac{m}{h}$	

Therefore, from II

$$\left(\frac{m}{h}\right) \times 5 = d$$

$$\frac{5m}{h} = d$$

Please proceed to question 8 below.

114
2

Question 8

Apply your knowledge and select the letter next to the correct answer to the following question: "An automobile went up the hill at a speed of 10 miles per hour and came down the same hill at a speed of 20 miles per hour. What was the average speed for the round trip?"

(A) $12\frac{1}{2}$ miles per hour

(B) $13\frac{1}{3}$ miles per hour

(C) $14\frac{1}{2}$ miles per hour

(D) 15 miles per hour

Very good. This problem was a tricky one. Let's review one method that could have been used to solve the problem:

I: THE VARIABLES:
We don't know what his rate was going; let's call it x.
The rate returning, however, was twice as fast; therefore, it can be designated as $2x$.
In each case the distance was the same, 120 miles.

Let's put this information in a CHART:

	R	T	D
Going:	x	(120)	120
Coming:	$2x$	($\frac{120}{2x}$)	120

Since $T = \frac{D}{R}$, T (going) $= \frac{120}{x}$

T (coming) $= \frac{120}{2x}$

II: THE RELATIONSHIP:
We are told that the total time is $4\frac{1}{2}$ hours.

i.e. T (going) + T (coming) $= 4\frac{1}{2}$

$\frac{120}{x} + \frac{120}{2x} = \frac{9}{2}$ ∝ $\cdot 2x$

$240 + 120 = 9x$ [C ^ A

$360 = 9x$ ∝ ÷ 9

$40 = x$ (rate going)

$80 = 2x$ (rate coming)

III: THE CHECK:

	R	T	D
Going:	(40)	(3)	120
Coming:	(80)	($1\frac{1}{2}$)	120

and Total Time: ($4\frac{1}{2}$)

Please proceed to question 17 below.

Question 17

Apply your knowledge and solve the following problem:

" A boy hiked to a lake at the rate of 2 miles per hour. He returned on a bicycle at the rate of 10 mph. If the entire trip took three hours, how far did he walk? "

Select the letter next to the correct answer.

(A) 25 miles

(B) $2\frac{1}{2}$ miles

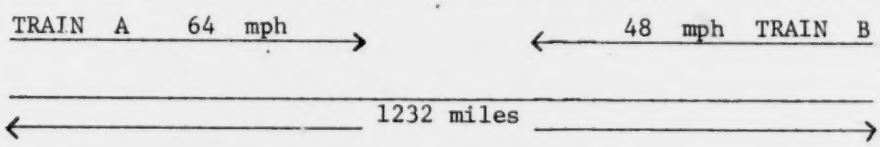
(C) 5 miles

(D) 12 miles

It is important that you develop a procedure for solving verbal problems. Let us use this problem as a means of outlining a working plan.

SKETCH:

Read the problem and make a sketch describing the situation:



Observe that the trains are 1232 miles apart, and that they are traveling towards each other at rates of 64 mph and 48 mph.

I: THE VARIABLES:

Read the problem again. This time you are concerned with what you are trying to find. In this case, you are asked to find the time it takes for the two trains to meet. You also note that both trains travel for the same length of time. Since this is a motion problem you recall the basic rule:

" RATE X TIME = DISTANCE "

The CHART:

All of this information is now organized into a chart form:

	RATE	TIME	DISTANCE
TRAIN A	64	x	64x
TRAIN B	48	x	48x

II: THE RELATIONSHIP:

You read the problem once more in order to determine the relationship between the information entered on the chart. " The total distance equals the sum of the two distances. " This will allow you to write the equation:

$$64x + 48x = 1232 \qquad [\ C\ \hat{}\ A$$
$$112x = 1232 \qquad \propto \div 112$$
$$x = 11$$

III: THE CHECK:

Set up a second chart entering the newly found values in place of the variables. Verify the relationships as you re-read the problem once more.

	RATE	TIME	DISTANCE
A	64	11	704
B	48	11	528
TOTAL			1232

Please go on to question 10 on page 117/1 .

Question 10

Apply the techniques of problem solving to this exercise:

" Train F starts from New York City and travels at the average rate of 60 mph towards Chicago, a distance of 975 miles. At the same time, Train G leaves Chicago and travels toward New York on a parallel track. If the two trains pass each other in 8 hours and 20 minutes, what was the average speed of the second train? "

Select the letter next to your answer.

(A) 48 mph.

(B) 57 mph.

(C) The same as Train F

(D) It can't be found.

If both bins contain 150 pounds of candy and the contents of one bin is denoted by x , the contents of the second bin is

(150 - x)

Recall the following: If the sum of two quantities is given, and one of the quantities is denoted by x , the other quantity is denoted by the sum minus x .

Please continue.

Return to page 112/2 and try this question again.

118
1

Very good. You made the correct choice.

I: THE VARIABLES:

Start by letting

x = the rate of the slower train

$x + 9$ = the rate of the faster train

The chart of the given facts is as follows:

Rule:	R	x	T	=	D
Slower Train	x		6		6x
Faster train	x + 9		6		6 (x + 9)

II: THE RELATIONSHIP:

The distance covered by both trains is the distance between the two towns, which is 726 miles. We have the equation,

$6x + 6(x + 9) = 726$ [D

$6x + 6x + 54 = 726$ [C^A

$12x + 54 = 726$ ∝-54

$12x = 672$ ∝÷12

$x = 56$

then $x + 9 = 65$

III: THE CHECK:

	R	T	D
Slower Train	(56)	6	(336)
Faster train	(65)	6	(390)
Together			(726)

Please go on to question 15 below.

118
2

Question 15

Apply your knowledge and solve the following problem:

"Mr. X traveled a distance of 240 miles in 5 1/2 hours. He went 50 mph part of the time and 40 mph the rest of the time. How long did he travel at 50 mph?

Select the letter next to the correct answer.

(A) More than 1 1/2 hours, but less than 2 1/2 hours.
(B) 3 1/2 hours
(C) 2 1/2 hours
(D) None of these.

Very good. You made the correct choice.

$$(6 > 5\tfrac{1}{2}) \quad \text{and}$$

$$(6 < 6\tfrac{1}{4})$$

I: THE VARIABLES:
Let x = the time it will take for the two cars to be ten miles apart.
The following is a chart of the given facts:

Rule	R	x	T	=	D
Car I	30		x		30x
Car II	35		x		35x

II: THE RELATIONSHIP:
Since the two cars end up 10 miles apart, the combined distance they have to cover is 390 miles. We, therefore, have the equation:

$$30x + 35x = 390 \qquad [\ C \wedge A$$
$$65x = 390 \qquad \propto \div 65$$
$$x = 6$$

III: THE CHECK:

	R	T	D
I	30	(6)	(180)
II	35	(6)	(210)
TOGETHER:			390

390 is 10 less than 400

Please go on to question 12 below.

Question 12

Apply your knowledge and solve the following problem:

" A freight train running at the rate of 20 mph leaves a station three hours before an express train, which travels in the same direction at 50 mph on a parallel track. How long will it take the express to catch the freight train? "

Select the letter next to the correct answer.

(A) 5 hours

(B) Less than 5 hours, but more than 3 hours

(C) Less than 3 hours

(D) None of these.

Congratulations.

This is not an easy problem. Did you start by making a sketch of the situation? Let's do that now:

I: (a) ——————→ @ $3\frac{1}{2}$ mph

(b) ←—————— @ $3\frac{1}{2}$ mph

P Q

	R	T	D
(a)	$3\frac{1}{2}$	$\left(\frac{x}{3\frac{1}{2}}\right)$	x
(b)	$3\frac{1}{2}$	$\left(\frac{x}{3\frac{1}{2}}\right)$	x

Total Time:

$$\frac{x}{3\frac{1}{2}} + \frac{x}{3\frac{1}{2}}$$

II: (a) ——————→ @ 3 mph

(b) ←—————— @ 4 mph

P Q

	R	T	D
(a)	3	$\left(\frac{x}{3}\right)$	x
(b)	4	$\left(\frac{x}{4}\right)$	x

Total Time:

$$\frac{x}{3} + \frac{x}{4}$$

The Relationship: The Second trip took 5 minutes longer than the First.

Note:

$$\frac{x}{3} + \frac{x}{4} = \frac{x}{3\frac{1}{2}} + \frac{x}{3\frac{1}{2}} + \frac{1}{12} \qquad \left(\frac{1}{3\frac{1}{2}} = \frac{1}{\frac{7}{2}} = \frac{2}{7} \right)$$

$$\frac{x}{3} + \frac{x}{4} = \frac{2x}{7} + \frac{2x}{7} + \frac{1}{12} \qquad \times 84$$

$$28x + 21x = 24x + 24x + 7 \qquad [\ C \wedge A$$

$$49x = 48x + 7 \qquad \times -48y$$

$$x = 7$$

Answer

I	R	T	D
(a)	$3\frac{1}{2}$	(2)	(7)
(b)	$3\frac{1}{2}$	(2)	(7)

Total Time 4 hours.

II	R	T	D
(a)	3	$(\frac{7}{3})$	(7)
(b)	4	$(\frac{7}{4})$	(7)

Total Time:

$$\frac{7}{3} + \frac{7}{4} =$$

$$\frac{28}{12} + \frac{21}{12} =$$

$$\frac{49}{12} =$$

$$4\ \frac{1}{12} \text{ hours.}$$

Therefore, II is $\frac{1}{12}$ hour (or 5 minutes) longer. ✓

Please go on to page $\frac{121}{1}$.

You have now finished this Segment. Hand in your PUNCH CARD. You should now have entered in your notebook the following definitions and formulas:

$$\text{Distance} = \text{Rate} \times \text{Time}$$

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT:

Problems 9, 10 , 11 , 12

We do not agree.

You are given the value of one gallon of the cheaper oil and of one gallon of the more expensive oil. Now, if one gallon costs 40¢ two gallons cost twice as much, three gallons cost three times as much, and x gallons cost x times as much. Thus, the value of x gallons of oil at 40¢ per gallon is $40x$.

Place the given facts in a chart. The headings could be

(# gals.) x (¢/gal) = (#¢)

Please return to page $\frac{138}{2}$ and try this question again.

122/1

After obtaining an answer algebraically, you should examine this answer from a common sense point of view. Now, a twelve mile hike to the lake would take the boy six hours. Since we are only allowed three hours for the round trip, this answer must be rejected. Subject your answer to a common-sense examination; it will help you avoid choosing impossible results.

Return to page 115/2 and try this question again.

122/2

Since the value of the nickels is \$1.05 , the first question you should answer is:

How many nickels have this value?

Use a chart for organizing the facts and relationship.

Follow this sequence of operations:

$$\left(N = \frac{V}{R} \right)$$

(1) Find the number of nickels.

(2) Use this to find the remainder of the 26 coins which would be the number of quarters.

(3) Find the value of that number of quarters.

Please return to page 136/2 and try this question again.

Very good. You made the correct choice.

I. THE VARIABLES:

We start by

letting x = the time Mr. X traveled at 50 mph,

$5\frac{1}{2} - x$ = the time Mr. X traveled at 40 mph.

Note that if the entire trip took $5\frac{1}{2}$ hours, and part of this time is denoted by x, the rest of the time is denoted by the total time minus x.

We can now fill in the chart:

Rule:	R	x T	= D
1st Part	50	x	$50x$
2nd Part	40	$\frac{11}{2} - x$	$40\ (\frac{11}{2} - x)$

II: THE RELATIONSHIP:

Since the total distance traveled is 240 miles, we have the equation,

$$50x + 40\ (\tfrac{11}{2} - x) = 240 \quad [\ D$$
$$50x + 220 - 40x = 240 \quad [\ C\ \wedge\ A$$
$$220 + 10x = 240 \quad \propto -220$$
$$10x = 20 \quad \propto \div 10$$
$$x = 2$$

III: THE CHECK:

	R	T	D
I	50	(2)	(100)
II	40	$(\frac{7}{2})$	(140)
TOTAL	-	$(5\frac{1}{2})$	(240)

Please go on to question 16 below.

Question 16

Apply your knowledge and solve the following problem:

"A man rode 120 miles in one train, then returned in another train the same route going twice as fast. The time for the round trip was $4\frac{1}{2}$ hours. How fast did each train travel?"

Select the letter next to the correct answer.

(A) 35 and 70 mph
(B) The sum of the speeds of both trains is 110 mph
(C) The faster train traveled at more than 75 mph
(D) The slower train traveled at 38 mph.

124
1

Very good. You made the correct choice.

I: THE VARIABLES:

We start by

letting x = the number of hours it will take the express to catch the freight.

Since the freight train has a three hour head start, we

let $x + 3$ = the time the freight train will travel.

Rule	R	x	T	=	D
Freight	20		$x + 3$		$20(x + 3)$
Express	50		x		$50x$

II: THE RELATIONSHIP:

When trains are together, the distances traveled by each are equal. Thus, we have the equation,

$$20(x + 3) = 50x \qquad [\ D$$
$$20x + 60 = 50x \qquad \propto -\ 20x$$
$$60 = 30x \qquad \propto \div\ 30$$
$$2 = x$$

III: THE CHECK:

The freight train travels for five hours at 20 mph = 100 miles. The express train travels for two hours at 50 mph = 100 miles. Therefore, the express caught up with the freight train.

Please go on to question 13 below.

124
2

Question 13

Apply your knowledge and solve the following problem:

"Achilles and the tortoise have a race. Achilles gives the tortoise a handicap of 990 yards. Achilles runs at the rate of 500 yards per minute, and the tortoise moves at the rate of 5 yards per minute? How long will it take Achilles to overtake the tortoise?"

(A) 2 hours

(B) $2\frac{1}{3}$ minutes

(C) $2\frac{1}{9}$ minutes

(D) More than $\frac{8}{9}$ minutes, but less than $2\frac{1}{3}$ minutes.

$\frac{125}{1}$

We do not agree.

If the sum of two quantities is given and one of the quantities is denoted by x, the second quantity is the sum minus x. Also note that value is equal to cost per unit $\times$ number of units.

Please return to page $\frac{112}{2}$ and try this question again.

$\frac{125}{2}$

Your answer does not check.

If the mixture contains eight quarters, the value of the quarters is $2.00. Since there are 28 coins in all, there must be 20 dimes in the mixture. The value of the dimes is also $2.00. Hence, the total value of the mixture is $4.00.

Please return to page $\frac{132}{2}$ and try this question again.

Very good. You made the correct choice.

I: THE VARIABLES:

One way to start is by letting

x = the time it took the boy to hike to the lake;

$3 - x$ = the time it took to return by bicycle.

(Since the total trip took three hours.)

We put these facts into a chart as follows:

Rule:	R	x	T	=	D
Walking:	2		x		2x
Riding:	10		3 - x		10 (3 - x)

II. THE RELATIONSHIP:

Since the trip going is equal in distance to the trip returning, we have the equation:

$$2x = 10(3 - x) \qquad [\ D$$
$$2x = 30 - 10x \qquad \propto +\ 10x$$
$$12x = 30 \qquad \propto \div\ 12$$
$$x = 2\tfrac{1}{2} \text{ hours}$$

The distance to the lake is

$$2\left(2\tfrac{1}{2}\right) = 5 \text{ miles}$$

III: THE CHECK:

R	T	D
2	$\left(2\frac{1}{2}\right)$	(5)
10	$\left(\frac{1}{2}\right)$	(5)

It is interesting to note that there are other approaches to the correct solution. Suppose, for example, you let

x = the distance.

(It is a reasonable substitution since the distance is what the problem is seeking.) We will let the CHART speak for itself:

	R	T	D				
Walking	2	$\frac{x}{2}$	x	$\frac{x}{2} + \frac{x}{10}$	=	3	$\propto 10$
Riding	10	$\frac{x}{10}$	x	$5x + x$	=	30	[C^A
Both		3		$6x$	=	30	$\propto \div 6$
				x	=	5	

(which is the distance sought)

Please go on to page 127/1 for the next question.

Question 18

Apply your knowledge and solve the following problem: "Starting at P, a man walks to Q at 3 mph and back at 4 mph. This entire trip takes five minutes longer than the time required to travel from P to Q and back at the rate of 3½ mph. What is the distance between P and Q ?"

Select the letter next to the correct answer.

(A) 11 miles

(B) 7 miles

(C) 84 miles

(D) There is no difference in time between both trips.

It seems that you analyzed the problem correctly in all respects except one. The original mixture contained

(5) (.20)

quarts of anti-freeze, but when three quarts of water were added, it did add to the total number of quarts of the solution, although it did not add to the total number of quarts of anti - freeze.

Evidently you felt that the new eight quart mixture contained four quarts of anti-freeze since three quarts were added (but it was only water) for in that case your answer would be correct.

Please return to page 146/2 and reconsider the analysis of this problem.

128
1

Very good. You made the correct choice.

I: THE VARIABLE:

One gallon of the cheaper oil is 40¢ , and the value of x times as much, is 40x .

The value of one gallon of the more expensive oil is 85¢ , and the value of (65 - x) such gallons is 85 (65 - x) .

II: THE RELATIONSHIP:

Hence, the total value is the sum of the separate values.

Total value = 40x + 85 (65 - x)

(# gals.) (¢/gal.) (# ¢)

	N	R	V
A	x	40	40 (x)
B	(65 - x)	85	85 (65 - x)
TOTAL			(40x) + 85 (65 - x)

Please go on to question 4 below.

128
2

Question 4

Apply your knowledge and select the letter next to the correct answer to the following:

If x pounds of coffee costing 85¢ a pound is mixed with 50 pounds of coffee worth 98¢ a pound, what is the value of the mixture?

(A) 85x + 98 (50)

(B) 85 (50) + 98x

(C) 50x + 85 (90)

(D) 98 (50 - x)

We do not agree.

One of the letters does have the correct answer next to it.

Find the number of nickels, then the number of quarters and finally their value in that order.

Return to page 136/2 and try this question again.

In mixture problem where two different solutions are described completely, that is, where the N and the R are given and you are asked to find the percent concentration of the resulting mixture, there are three things to keep in mind:

(1) The sum of the N of each solution equals the N of the mixture

(2) The sum of the V of each solution equals the V of the mixture.

(3) But the percentages, the R of each cannot be added to produce the R of the mixture. It is actually somewhere between the two R's of the solutions.

To find the R of the mixture, you should use the basic rule:

$$R = \frac{\text{Total V}}{\text{Total N}}$$

VOLUME 6 SEGMENT 4 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	2 9	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	56	0 6	(Volume Number)
	66	and	68	0 4	(Segment Number)

Your reading assignment for this Segment is pages 182 - 183 .

SUPPLEMENTARY NOTES:

The problems that you will meet in this segment are known as "Cost Problems," or "Mixture Problems" It doesn't really matter whether the discussion is about "candy" "nuts," "coffee," "chemical compounds," "seeds" or any other commodity for the problems all have the same fundamental underlying principle. This principle involves three elements:

(1) The number of units, N (e.g. 3 pounds, 5 gallons, etc.)

(2) The rate per unit, R (e.g. $2 per pound, 29¢ per gallon, etc.) This is also referred to as the "price," "unit cost," "the worth per unit," the % concentration, etc. all of which are basically the same thing as the "rate."

(3) The total value of the whole batch, V.

These three elements are related by the rule:

(THE NUMBER OF UNITS) x (THE RATE PER UNIT) = (THE TOTAL VALUE)

or N x R = V

You will, no doubt recognize the similarity between this rule and the rule that you have just worked with in the previous segment in "uniform motion problems."

(Rate) x (Time) = (DISTANCE)

R x T = D

Actually, algebraically, the two rules are equivalent. The same relationships exist between the elemements of the rules: e. g.

just as $\frac{D}{R} = T$ so too, $\frac{V}{R} = N$

also $\frac{D}{T} = R$ while $\frac{V}{N} = R$

Please go on to the next page.

Question 1

Apply the proper principles and find the value of 80 pounds of coffee if a 100 pound bag has a value of $65 .

Select the letter next to the correct answer.

(A) $62

(B) $52

(C) $48

(D) None of these.

There are two kinds of coffee considered here. One kind is worth 85¢ a pound and the other kind is worth 98¢ a pound. The value of each of these coffees is the product of the cost per pound and the number of pounds. Use a chart to organize this information and observe the relationships.

Return to page 128/2 and try this question again.

132
1

Very good. You made the correct choice.

I: THE VARIABLE:

	N	R	V
(1) Nickels	(21)	.05	1.05
(2) Quarters	(5)	.25	5(.25)
Total	26		

$(N = \frac{V}{N} \quad \frac{1.05}{.05} = 21)$

(No. of quarters = 26-No. of nickels

= 26 - (21)

= (5)

(3) Value of quarters = (5) (.25)

= $1.25

(1) We are told that the value of the nickels is $1.05 . This means that there are 21 nickels.

(2) Thus, there are five quarters in the mixture since the total number of coins is 26 .

(3) The value of five quarters is $1.25 .

Please go on to question 6 below.

132
2

Question 6

Apply your knowledge and select the letter which correctly completes the statement:

A mixture of 28 coins consisting of dimes and quarters has a value of $4.10 ; then,

(A) There are 8 quarters.

(B) There are 21 dimes.

(C) There are 16 quarters.

(D) The given mixture cannot have a value of $4.10.

Value is equal to the product of the price of one pound and the number of pounds involved. The price of one pound is given as 95¢ . However, the number of pounds is not

$$(80 - x)$$

Keep in mind that the total number of pounds in both bins is 150 .

Please return to page 112/2 and try this question again.

We do not agree. In order to keep track of all the given data, it is important to make a chart as follows:

I: THE VARIABLES:

	# lbs. coffee		¢/lb.		¢ Value
Rule	N	x	R	=	V
Kind 1					
Kind 2					
Mixture					

Under the N column, you fill in the two amounts that comprise the mixture, and the amount of the mixture. Under the R column, you fill in the prices (in cents) of each of the amounts that are mixed and the price of the mixture. The rule on the top line of the chart tells you how to fill in the third column.

II: The RELATIONSHIP:

An equation is obtained by noting that the value of the mixture is equal to the sum of the values of the kinds that were mixed together.

Please return to page 144/2 and try this question again.

134/1

You have made some mistake in analyzing this problem.

Why don't you set up the chart with these headings:

	# qts. N	% antifreeze R	qts. antifreeze V
Solution:			
Water:			
Mixture:			

Consider the fact that to find the percent of antifreeze in the mixture, you must find the N and the V of the mixture and then apply the rule,

$$R = \frac{V}{N}$$

Please return to page 146/2 and continue with this problem.

134/2

Your answer does not check. 60 pounds of grade A is worth

$$60 \times .96 = \$57.60$$

40 pounds of grade B is worth

$$40 \times .84 = \$33.60$$

The sum of these values is

$$\$91.20$$

The value of the mixture is

$$100 \times .92 = \$92.00$$

Please return to page 147/2 and try this question again.

Did you check your answer? Twenty pounds of cashews have a value of

$$20 \times \$.49 = \$9.80$$

and 40 pounds of peanuts have a value of

$$40 \times .31 = \$12.40$$

The combined value of these two does not equal the value of the mixture,

$$60 \times \$.43$$

Please return to page 151/1 and try this question again.

We do not agree.

Note that you are given the value of 90 gallons of gas . It is not necessary to find the unit price of this gas. The fact that one box in your chart is empty should not worry you.

If we let

x = the number of gallons of the cheaper gas, then

$28x$ = the value of this gas.

How many gallons are there in the mixture? Yes,

$(90 + x)$ gallons

Please continue.

Please return to page 149/2 and try this question again.

136
1

Very good. You made the correct choice.

I: THE VARIABLES:

The mixture consists of x pounds of coffee and 50 pounds of coffee. The x pounds cost 85¢ a pound. Thus, the value of this coffee is 85x . The 50 pounds of coffee costs 98¢ a pound.

(# lbs. coffee) x (¢ per lb.) = (total # ¢)

	N	R	V
A	x	85	85 (x)
B	50	98	98 (50)
Together			85 (x) + 98 (50)

The value of this coffee is, therefore,

98 x 50

II: THE RELATIONSHIP:

Together, the two coffees have the value

85x + 98 (50)

Please go on to question 5 below.

136
2

Question 5

Apply your knowledge and select the letter next to the correct answer to the following question:

You have a mixture of nickels and quarters consisting of 26 coins. If the value of the nickels is $1.05 , what is the value of the quarters?

(A) $4.75
(B) $1.25
(C) $2.25
(D) None of these.

Did you try this problem algebraically?

Start by letting

x = the number of quarters;

$28 - x$ = the number of dimes.

In terms of dollars, the value of the quarters is

$.25x$

and the value of the dimes is

$.10\ (28 - x)$

Please continue.

Please return to page 132/2 and try this question again.

We do not agree. The equations obtained in solving mixture problems sometimes involve large numbers. You may have made an error in finding the solution set of the equation. Let us solve a <u>similar</u> equation together.

Find the solution set of

$$115x + 3300 = 110\ (40 + x) \qquad [\ D$$

$$115x + 3300 = 4400 + 110x \qquad \text{<-}110x$$

$$5x + 3300 = 4400 \qquad \text{<-}3300$$

$$5x = 1100 \qquad \text{<-}\div 5$$

$$x = 220$$

Please return to page 157/1 and try this question again.

138
1

Very good. You made the correct choice.

I: THE VARIABLES:

Both bins contain 150 pounds of candy. One of the bins contains x pounds, therefore, the other bin contains

$$(150 - x) \text{ pounds}$$

Now, the value is equal to the product of the unit price and the number of pounds. Thus, the value equals

$$95\ (150 - x)$$

(No. of lbs.) (¢ per lb.) (value in ¢)

	N	R	V
(1)	x	80	(80x)
(2)	(150 - x)	95	95 (150 - x)

II: NOTE: No relationship was given connecting the values of the contents of the two bins. Until that is given, the problem can be analyzed no further.

Please go on to question 3 below.

138
2

Question 3

Apply your knowledge and select the letter which represents the total value of the following mixture:

x gallons of 40¢ oil and
(65 - x) gallons of 85¢ oil.

(A) 40x + 85 (65 - x) cents

(B) x (105) cents

(C) 40x + 65 (85 - x) cents

(D) 65 (85 + 40) cents

Very good. You made the correct choice. Let's review a procedure for handling a problem of this sort.

I: THE VARIABLES:

Let x be the per cent anti-freeze of the new mixture. Place the given facts in chart form. The entries in parentheses are derived figures.

	N	R	V	
Solution:	5	.20	(5) (.20)	
Water:	3	0	(0)	(The water doesn't contain any anti-freeze.)
MIXTURE:	8	x	(5) (.20)	

II: THE RELATIONSHIP:

The total mixture contains the sum of the two amounts of anti-freeze.

	N R	=	V	
The equation	8 x	=	(5) (.20)	
	8 x	=	1.00	$\div 8$
	x	=	$.12\frac{1}{2}$	(change hundredths to %)
	x	=	$12\frac{1}{2}\%$	

You will note that we follow the same general procedure for this type problem as we do for all mixture problems, even though some of the entries are zero.

Please proceed to question 8 below.

Question 8

Apply your knowledge and select the letter which correctly fills in the missing number in the following statement: If eight quarts of a 25% solution of iodine are mixed with two quarts of a 50% solution of iodine, the result is ten quarts of a_____% solution of iodine."

(A) 30
(B) 35
(C) 40
(D) 75%

$\frac{140}{1}$

We do not agree.

It is possible that you made an error in finding the solution set of the equation. If we let

$$x = \text{the number of pounds of peanuts,}$$

then the value (in cents) of the peanuts is $31x$; and the value of the cashews is

$$49 (60 - x) ¢$$

The value of the mixture is

$$60 (43) ¢$$

Please go over your multiplication.

Return to page $\frac{151}{1}$ and try this question again.

$\frac{140}{2}$

We do not agree.

If you check your answer, you will discover that your choice is not correct.

Start by letting

x = the number of pounds of grade A coffee, and

$(100 - x)$ = the number of pounds of grade B coffee.

Make a chart of the given facts and continue.

Return to page $\frac{147}{2}$ and try this question again.

You should check your choice. Note that 16 quarters have a value of $4.00 . Now, there are still 12 dimes to be considered. The total value is much larger than $4.10 .

Please return to page 132/2 and try this question again.

Please go over your work and check the multiplication and division involved in finding the solution set of the equation you used. One of the other answers is the correct choice.

Please return to page 149/2 and try this question again.

142/1

Your answer does not check.

Keep in mind that the sum of the values of each type of seed must be equal to the value of the mixture. Now, 90 pounds of seed at $1.05 a pound has a value of

$$90 \times 105 = \$94.50$$

30 pounds of seed at 90¢ per pound has a value of $27.00 . The sum of these values is $121.50 . This should equal

$$100\ (120) = \$120.00$$

Since the two values are not the same, your answer is incorrect.

Please return to page 157/1 and try this question again.

142/2

We don't agree. If we let

x = the number of quarts of the 60¢ oil,

the chart of the given facts is as follows:

	N	R¢	V¢
1	x	60	60x
2	(56 - x)	54	54 (56 - x)
Mix	56	56	V

Now it will be possible to construct an equation from this information if we can find two different expressions that describe the same value. We can. The value V of the mixture can be described in two ways:

(1) (The Value of ingredient A) + (The Value of ingredient B) = total Value of Mix

(2) (The Number of units of the mixture) x (unit price) = total Value of Mix

Please return to page 155/1 and finish the problem.

Did you wonder why there is a difference of 5¢ between selling the lemon drops separately and selling them together? It seems reasonable that an equal amount at three for 1¢ mixed with an equal amount at two for 1¢ should bring in the same money as both amounts of lemon drops selling five for 2¢ . But, there is a difference. Mathematically we can say that the average of the prices per lemon drop, $\frac{1}{3}$ ¢ and $\frac{1}{2}$ ¢ is not $\frac{2}{5}$ ¢ .

In other words,

$$\frac{\frac{1}{3} + \frac{1}{2}}{2} \quad \text{does not equal} \quad \frac{2}{5}$$

However, the above only explains why there is a difference; it does not. help us solve this problem. Start by letting

x = the number of lemon drops in each of the boxes.

Find the cost of each single lemon drop.

	N	R	V
if (A)	3		1¢
then		R = $\frac{1}{3}$ ¢	

	N	R	V
if (B)	2		1¢
then		R = $\frac{1}{2}$ ¢	

When sold separately,

	N	R	V
(A)	x	$\frac{1}{3}$	$\frac{x}{3}$

	N	R	V
(B)	x	$\frac{1}{x}$	$\frac{x}{2}$

If each box is sold separately, the sales will total

$$\frac{x}{3} + \frac{x}{2}$$

If the two boses are sold together, the sales will total

$$2x \left(\frac{2}{5} \right)$$

Please continue by comparing the two total values.

Please return to page $\frac{165}{1}$ and try this question again.

144
1

Very good. You made the correct choice.

I: THE VARIABLES:

	# Qts.	% Iodine	# Qts. Iodine	Calculation
	N	R	V	
A	8	.25	(.25) (8)	(.25) (8) = 2
B	2	.50	(.50) (2)	(.50) (2) = 1
Both	10	x	(3)	Sum = 3

We first find the amount of pure iodine in each of the solutions:

(A) Eight quarts of the 25% solution of iodine contains two quarts of pure iodine.

(B) Two quarts of the 50% solution contains one quart of iodine.

II: THE RELATIONSHIP:

The mixture of 10 quarts contains three quarts of pure iodine.

$$10x = 3 \qquad \text{«} \div 10$$

$$x = \frac{3}{10}$$

$$x = 30\%$$

Please go on to question 9 below.

144
2

Question 9

Apply your knowledge and solve the following problem:

"How many pounds of coffee worth 72¢ a pound must be added to 75 pounds of coffee worth 80¢ a pound to make a mixture to sell at 76¢ a pound?"

Select the letter next to the correct answer.

(A) 150 pounds

(B) Less than 80 pounds, but more than 72 pounds.

(C) More than 75 pounds, but less than 80 pounds.

(D) 155 pounds

The total mixture contains 120 pounds. If we let

x = the number of pounds of peanuts, then

$2x$ = the number of pounds of cashews.

Now, how do we represent the number of pounds of almonds in the mixture? Yes, the number of pounds of almonds is represented by

$$(120 - 3x)$$

The peanuts plus the cashews amount to $3x$ pounds; therefore, the number of pounds of almonds is equal to the total minus $3x$.

Please continue.

Return to page 158/2 and try this question again.

A monomial is a phrase that is either a variable or a constant, or the product of a constant and one or more variables. Therefore, $\frac{2}{3}z$ and $.07w^2$ are monomials. But you were asked to choose phrases that are not monomials.

Return to page 160/2 and try this question again.

Very good. You made the correct choice.

I: THE VARIABLES:

We start by letting

x = the number of quarters

	N	R	V
Q	x	.25	(.25x)
D	(28 - x)	.10	(.10) (28 - x)
Both	28		4.10

II: THE RELATIONSHIP:

Since the total value of the mixture is $4.10 , we can write the equation,

Value of Quarters	+	Value of Dimes	=	$4.10	
.25x	+	.10 (28 - x)	=	4.10	∝·(100)
25x	+	10 (28 - x)	=	410	[D
25x	+	280 - 10x	=	410	[C^A
15x	+	280	=	410	∝- 280
15x			=	130	∝÷ 15
x	=	$\frac{130}{15}$	=	$8\ \frac{10}{15}$	

III: THE CHECK:

Since it is not possible to have a fractional number of coins, the mixture cannot have a value of $4.10 .

Please go on to question 7 below.

Question 7

Apply your knowledge and select the letter which correctly answers the following question:

" If five quarts of a 20% solution of antifreeze is added to three quarts of water, what percent solution of antifreeze is the resulting mixture?"

(A) 50%
(B) 20%
(C) 15%
(D) $12\ \frac{1}{2}$%

Very good. You made the correct choice.
The answer 60 lies between 55 and 65 .

I: THE VARIABLES:
We start by letting

x = the number of pounds of the $1.05 seed

and make a chart of the given facts:

	N	x	R¢	=	V¢
A	x		105		105x
B	30		90		90(30)
MIX	x + 30		100		100(x + 30)

II: THE RELATIONSHIP:
Since the value of the mixture is equal to the sum of the values of each kind, we have the equation:

$$105x + 90(30) = 100(30 + x) \quad [\text{ D}$$
$$105x + 2700 = 3000 + 100x \quad \text{<-100x}$$
$$5x + 2700 = 3000 \quad \text{<-2700}$$
$$5x = 300 \quad \text{<÷5}$$
$$x = 60$$

III: THE CHECK:

N		$R		$V
60	x	$105	=	$63.00
30	x	$.90	=	$27.00
90		$1.00		$90.00
90	x	$1.00	=	$90.00

$63. + $27. = $90.00 (and) ✓

Please go on to question 12 below.

Question 12

Apply your knowledge and solve the following problem: Grade A coffee worth 96¢ a pound is mixed with Grade B coffee worth 84¢ a pound to yield 100 pounds of coffee that will sell at 92¢ a pound. How many pounds of each kind were used? Select the letter next to the correct answer.

	Pounds of Grade A	Pounds of Grade B
(A)	60	40
(B)	$63\frac{1}{3}$	$33\frac{2}{3}$
(C)	52	48
(D)	$66\frac{2}{3}$	$33\frac{1}{3}$

148/1

We do not agree. The phrase $2a^2$ is a monomial. It consits of the product of the constant 2 , the variable a , and the variable a . That is,

$$2a^2 = 2 \cdot a \cdot a$$

Return to page 166/2 and try this question again.

148/2

You evidently added the terms without considering whether or not they were similar.

Please note the following rule:

To add two polynomials, combine their similar terms.

One procedure is to use the COMMUTATIVE PROPERTY OF ADDITION and change the order of the terms so that similar terms are next to each other. Then by the ASSOCIATIVE PROPERTY, the sums of the various sets of similar terms when combined into one polynomial will be the sum of the original polynomials.

Return to page 177/2 and try this question again.

$\frac{149}{1}$

Very good. You made the correct choice.

I: THE VARIABLES:

We start by letting

x = the number of quarts of the 60¢ oil;

$56 - x$ = the number of quarts of the 54¢ oil.

The chart of the given facts appears below.

	N	x	R¢	=	V¢
1	x		60		60x
2	56 - x		54		54(56 - x)
MIX	56		56		

II: THE RELATIONSHIP:

We have to find find the solution set of the equation,

The sum of the values		=	value of the mix.	
60x + 54(56 - x)		=	56(56)	[D
60x + 3024 - 54x		=	3136	[C ^ A
6x + 3024		=	3136	∝ -3024
6x		=	112	∝ ÷6
x		=	$18\frac{2}{3}$	
	56 - x	=	$37\frac{1}{3}$	

III: THE CHECK:

	N	$R	$V	
1	$(18\frac{2}{3})$	$.60	$11.20	$(18 + \frac{2}{3})60 = 1080 + 40$
2	$(37\frac{1}{3})$	$.54	$20.16	$(37 + \frac{1}{3})54 = 1998 + 18$
M	56	$.56	$31.36	

56 x $.56 = $11.20 + $20.16

Please go on to question 14 below.

$\frac{149}{1}$

Question 14

Apply your knowledge and solve the following problem:

A dealer has 90 gallons of gas worth $29.70 . He wished to mix this with gas worth 28¢ a gallon to produce a mixture worth 31¢ a gallon. How many gallons of the cheaper gas should he use?

Select the letter next to the correct answer.

(A) $6\frac{2}{3}$ gallons (C) 60 gallons

(B) $60\frac{2}{3}$ gallons (D) None of these.

Very good. You made the correct choice.

I: THE VARIABLES: We start by letting

x = the number of pounds of 72¢ coffee that must be added.

Next, we make a chart of the given facts as follows:

RULE:	# lbs. Coffee N	¢/lb. R	¢ Value V
	N	x R	= V
1 Kind	x	72	72x
2 Kind	75	80	75 (80)
Mixture	x + 75	76	76 (x + 75)

In the N column, we have x pounds, 75 pounds and the sum of these two. In the R column, we list the price of each kind and the price of the mixture. Note that since the column is headed " cents " we don't need the decimal. The value column is determined by the rule on the top line of the chart.

II: THE RELATIONSHIP: The equation that yields the solution set of the problem is obtained by equating the value of the mixture to the sum of the values of each kind. We have,

1st Value	+	2nd Value	=	Total Value	
72x	+	72 (80)	=	76 (x + 75)	[D
72x	+	6000	=	76x + 5700	∝ -72x
		6000	=	4x + 5700	∝ -5700
		300	=	4x	∝ ÷4
		75	=	x	

III: THE CHECK:

	N	x R	= V	
(1)	(75)	72	(5400)	
(2)	75	80	(6000)	
M	(150)	76	(11400)	11400¢ = $114.00

Note: If 75 lbs. of the first mixture is used, the value of the new mixture of 150 lbs. will be equal to the sum of the values of the separate mixtures.

Please go on to the next question on page 151 .
1

Question 10

Apply your knowledge and solve the following problem:

" If peanuts sell for 31¢ a pound, and cashews sell for 49¢ a pound, how many pounds of each is needed to make a 60 pound mixture that will sell for 43¢ a pound?

Select the letter next to the correct answer.

(A) 20 pounds of peanuts and 40 pounds of cashews.

(B) 20 pounds of cashews and 40 pounds of peanuts.

(C) 25 pounds of peanuts and 35 pounds of cahsews.

(D) None of these.

Did you make a chart of the facts? Your chart should look like this:

	N	x	R¢	=	V¢
Peanuts	x		30		
Cashews	2x		75		
Almonds	120 - 3x		120		
Mixture	120		75		

Please fill in the rest of the boxes and write the equation needed to solve this problem. Be sure to check the solution.

Please return to page 158/2 and try this question again.

Very good. You did a fairly difficult problem correctly. The unit price of the cheaper lemon drops is

$$\frac{1}{3}\ ¢$$

and the unit price of the more expensive lemon drops is

$$\frac{1}{2}\ ¢$$

If we let

x = the number of lemon drops in each of the boxes, then

$\frac{x}{3}$ = the value of the cheaper box of lemon drops

$\frac{x}{2}$ = the value of the more expensive box.

The sum of these two numbers is the total value of both boxes. Now, if both boxes are combined, we have $2x$ lemon drops selling at

$$\frac{2}{5}\ ¢$$

each. The value of the mixture is, therefore,

$$(\tfrac{2}{5})\quad (2x)\ =\ \frac{4x}{5}$$

When sold separately:

	N	R¢	V¢
(A)	x	$\frac{1}{3}$	$\frac{x}{3}$
(B)	x	$\frac{1}{2}$	$\frac{x}{2}$

Total Value $(\frac{x}{3} + \frac{x}{2})$

When sold together:

	N	R¢	V¢
MIX	$2x$	$\frac{2}{5}$ ¢	$\frac{4x}{5}$

Total Value = $(\frac{4x}{5})$

II: The Total Value = Total Value of Mix plus 5¢ .

$$\frac{x}{3} + \frac{x}{2} = \frac{4x}{5} + 5 \qquad \times 30$$

$$10x + 15x = 24x + 150 \qquad \text{C} \wedge \text{A}$$

$$25x = 24x + 150 \qquad -24x$$

$$x = 150$$

This answer is more than 120 and less than 180 .

III:

	N	R¢	V¢
(A)	150	$\frac{1}{3}$	50
(B)	150	$\frac{1}{2}$	75
Total Value:			\$1.25

	N	R¢	V¢
MIX	300	$\frac{2}{5}$	\$1.20

The Separate Total Value is 5¢ more than the Total Value of the mix.

Please go on to page $\frac{153}{1}$.

You have now finished this Segment. Hand in your PUNCH CARD. In your NOTEBOOK you have the following important points to be considered when you are solving verbal problems:

(1) Read the problem carefully several times to determine the given facts.

(2) Draw a diagram illustrating the physical situation.
Label all parts.

(3) I: THE VARIABLES:
Determine the variable that you should call x .
Relate the other variables in terms of x .

(4) Construct a CHART.
Head the chart with the rule that applies:

e.g.

Rate	x	Time	=	Distance
Number	x	Unit Price	=	Total Value

(5) II: THE RELATIONSHIP:
Determine from another reading of the problem a relationship that exists between the two sets of entires in the chart.
This will be the basis for formulating an equation.
Solve the equation indicating the operators employed at each step.

(6) Label the answer:
Make sure that the answer is in the proper form.

(7) III: THE CHECK:
Enter the newly found values on a duplicate chart.
Verify all relationships that exist within the chart and with the original problem.

Complete the HOMEWORK ASSIGNMENT AS FAR AS PROBLEM 16 .

We agree that $5\frac{x}{y}$ is not a monomial. In a monomial, we may not have division by a variable. However, we do not agree with your second choice.

Please return to page 160/2 and try this question again.

Very good. You made the correct choice.

I: THE VARIABLES:

We start as follows:

let x = the number of pounds of grade A coffee;

$100 - x$ = the number of pounds of grade B coffee.

Now, we make a chart of the given facts:

Rule:	N	x	R¢	=	V¢
A	x		96		$96x$
B	$100 - x$		84		$84(100 - x)$
MIXTURE	100		92		$92(100)$

II: THE RELATIONSHIP:

Since the value of both kinds of coffee is equal to the value of the mixture, we have the equation,

$96x + 84(100 - x) = 92(100)$ [D

$96x + 8400 - 84x = 9200$ [C ^ A

$12x + 8400 = 9200$ ∝ -8400

$12x = 800$ ∝ ÷12

$x = 66\frac{2}{3}$

$100 - x = 33\frac{1}{3}$

III: THE CHECK:

	N	x	$R	=	$V
A	$(66\frac{2}{3})$		$.96		$64.00
B	$(33\frac{1}{3})$		$.84		$28.00
M	100		$.92		$92.00

We will check in dollars

Please go on to the next question on page 155.
1

Question 13

Apply your knowledge and solve the following problem:

How many quarts of oil at 60¢ a quart and oil at 54¢ a quart will yield a mixture of 56 quarts of oil selling at 56¢ a quart?

Select the letter next to the correct answer.

(A) 19 quarts of 60¢ , 37 quarts of 54¢ .

(B) $18\frac{2}{3}$ quarts of 60¢ , $37\frac{1}{3}$ quarts of 54¢ .

(C) 28 quarts of 60¢ , 28 quarts of 54¢ .

(D) 27 quarts of 60¢ , 28 quarts of 54¢ .

The quotient of two algebraic phrases such as

$$\frac{x + y}{x - y} \quad , \quad \text{or} \quad \frac{x}{y}$$

where both the numerator and denominator contain variables is not a binomial.

Return to page 166/2 and try this question again.

Very good. You made the correct choice.

I: THE VARIABLES:

We start by letting

x = the number of pounds of peanuts needed;

$60 - x$ = the number of pounds of cashews needed.

We now make a chart of the given facts:

	N	x	R¢	=	V
1	x		31		31x
2	60 - x		49		49(60 - x)
MIX	60		43		60(43)

Note: we use the cents notation to avoid having to use decimals.

II: THE RELATIONSHIP:

Since the value of the peanuts plus the value of the cashews is equal to the value of the mixture, we have the equation

$$31x + 49(60 - x) = 60(43) \quad [\ D$$

$$31x + 2940 - 49x = 2580 \quad [\ C \wedge A$$

$$2940 - 18x = 2580 \quad \propto -2940$$

$$-18x = -360 \quad \propto \div (-18)$$

$$x = 20$$

Thus, the mixture consists of 20 pounds of peanuts and 40 pounds of cashews.

III: THE CHECK:

	N		$R		V
(1)	20	x	.31	=	$6.20
(2)	40	x	.49	=	19.60
(3)	60		.43		$25.80

Note: we can convert the cents to dollars in the check for clarity.

In problems of this type, an assumption is made that each item that is being mixed retains its rate of price per unit even after it is mixed. The two different rates then produce a "weighted average" rate somewhere between the two, and the mixture has a mathematically accurate price rate. It has nothing to do with a "sale price." It is just a combination of the same original prices.

Question 11

Apply your knowledge and solve the following problem:

How many pounds of seed worth \$1.05 a pound must be mixed with 30 pounds of seed worth 90¢ a pound to produce a mixture to sell for \$1.00 a pound?

Select the letter next to the correct answer.

(A) 30 pounds

(B) 90 pounds

(C) Less than 50 pounds

(D) More than 55 pounds, but less than 65 pounds.

The degree of a monomial is the sum of the degrees in each of the variables. Thus, the degree of

$$6x^4y \quad \text{is} \quad 5$$

The degree of the constant 6 is zero; the degree of x is the exponent 4 ; and the degree of y is 1. This yields a total of 5 .

Please return to page 171/2 and try this question again.

158
1

Very good. You made the correct choice.

I: THE VARIABLES:

Let

x = the number of gallons of the 28¢ gas.

The chart of the facts is:

Rule:	N	x	R¢	=	V¢
A	90				2970
B	x		28		28x
MIX	90 + x		31		31 (90 + x)

II: THE RELATIONSHIP:

We have the equation,
The Sum of the Values = The Value of the mix.

$$2970 + 28x = 31(90 + x) \quad \text{D}$$
$$2970 + 28x = 2790 + 31x \quad \text{<-28x}$$
$$2970 = 2790 + 3x \quad \text{<-2790}$$
$$180 = 3x \quad \text{<÷3}$$
$$60 = x$$

III: THE CHECK:

	N	$R	$V
A	90		$29.70
B	(60)	$.28	($16.80)
M	(150)	$.31	($46.50)

The arrows indicate that the relationship checks.

Please go on to question 15 below.

158
2

Question 15

Apply your knowledge and solve the following problem:

A mixture of peanuts, cashews, and almonds weighs 120 pounds. The mixture contains twice as many pounds of cashews as it does peanuts. The peanuts sell for 30¢ a pound; the cashews for 85¢ a pound; the almonds for $1.20 a pound; and the mixture must sell for 75¢ a pound. How many pounds of each kind of nut must be used?

Select the letter next to the correct answer.

	lbs. of peanuts	lbs. of cashews	lbs. of almonds
(A)	30	50	40
(B)	28	54	38
(C)	20	40	60
(D)	None of these.		

We do not agree. TRI is a prefix meaning three.

A trinomial is a phrase consisting of three monomials combined by addition or subtraction.

$$3\ abc$$

has three variables, but they are combined by multiplication.

Return to page 176/2 and try this question again.

Polynomials can be simplified if they contain similar terms. What do we mean by similar terms? Terms that have the same variables to the same power are called similar terms.

Thus, for example,

$$5x^2y^3 \quad \text{and} \quad 13x^2y^3$$

are similar terms.

Return to page 175/2 and try this question again.

160
1

VOLUME 6 SEGMENT 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying material that must be furnished by you, you are asked to punch out the following:

COLUMNS	48 and 50	3	0	(Sequence Number)
	54 and 56	0	4	(Type of Punch Card)
	60 and 62	0	6	(Volume Number)
	66 and 68	0	5	(Segment Number)

Your READING ASSIGNMENT for this Segment is pages 197 - 198 and 200 - 201 .

SUPPLEMENTARY NOTE:

In the last few segments you learned how to solve several kinds of problems by using algebraic phrases and equations. We want to increase our ability to solve more complicated problems. To do so, we must learn more about operations with algebraic phrases. The simplest phrase is a variable, a single letter. More complicated phrases combine a variable with a numeral, other variables, or both. The algebraic phrase that we will now study is called a polynomial.

You will now be asked a series of questions to draw your attention to the more important points.

160
2

Question 1

Recognize which of the following phrases are not monomials.

I: $\frac{5x}{y}$ II: $\frac{2}{3}z$ III: $.07w^2$ IV: $x + 1$

(A) II and III (C) II and IV

(B) I and III (D) I and IV

In order for a polynomial to be written in decreasing degree of a variable, the term of highest degree in that variable must be written first. Then the term containing that variable to the next lower degree must be written next, and so on. For example, the polynomial

$$a^4b + 3a^6 - a^2b^2 + 5$$

written in decreasing degree of the variable a is

$$3a^6 + a^4b + a^2b^2 + 5$$

Note that the first term has the variable a to the highest degree, the second term has a to the next lower degree and so on.

On the other hand, if the same expression were written in decreasing degree of the variable b, it would appear as

$$-a^2b^2 + a^4b + 3a^6 + 5$$

Please return to page 181/2 and try this question again.

Only similar terms can be combined. Thus, for example

$$18k - 13k + 4m = 5k + 4m$$

Please return to page 185/2 and try this question again.

$\frac{162}{1}$

What is the degree of

$$10xyz^3$$

The degree of a monomial is the sum of the degrees in each of the variables. The degree of the variable x is one since a letter written without an exponent means that this letter is raised to the one power. Similarly, the degree of y is one and the degree of the variable z is three. The coefficient 10 is of degree zero. Therefore, the sum of all the degrees is five.

Please return to page $\frac{171}{2}$ and try this question again.

$\frac{162}{2}$

You made a careless error;

$$-\frac{1}{4} + 1 \text{ does not equal } -\frac{3}{4}$$

A careless error which anybody is capable of making should convince you of the importance of checking an answer. You could have discovered your mistake if you did the following:

Choose any convenient values for the letters.

Let $a = 2$ $b = 3$, then:

$$\left(\frac{1}{2}a - \frac{1}{3}b - \frac{1}{4}\right) + (a + 1 + b) \stackrel{?}{=} \left(\frac{3}{2}a + \frac{2}{3}b - \frac{3}{4}\right)$$

$$\frac{1}{2}(2) - \frac{1}{3}(3) - \frac{1}{4} + (2) + 1 + (3) \stackrel{?}{=} \frac{3}{2}(2) + \frac{2}{3}(3) - \frac{3}{4}$$

$$1 - 1 - \frac{1}{4} + 6 \stackrel{?}{=} 3 + 2 - \frac{3}{4}$$

$$-\frac{1}{4} + 6 \stackrel{?}{=} 5 - \frac{3}{4}$$

$$5\frac{3}{4} \neq 4\frac{1}{4}$$

This doesn't check, and banning a mistake in arithmetic, it means your answer choice is wrong.

Please return to page $\frac{172}{2}$ and try question 10 again.

In order for a polynomial to be written in increasing degree of a variable, the first term must contain the variable of lowest degree, the next term must contain the variable of the next higher degree, and so on.

Thus, for example,

$$2 + p + p^3 + p^4 + p^6$$

is a polynomial written in increasing degree of the variable p .

Please return to page 170/2 and try this question again.

Please be more careful in combining signed numbers.

$-2y + 3y$ does not equal $-5y$ or $-y$

and

$-7 + 5$ does not equal -12 or $+2$

It might be better to try one operation at a time to avoid confusing yourself.

Please return to page 177/2 and try this question again.

Very good. "None of these" is the correct choice.

I: We start by letting

x = the number of pounds of peanuts used;

$2x$ = the number of pounds of cashews used;

$120 - 3x$ = the number of pounds of almonds used.

It is important to note that the number of pounds of almonds is denoted by the total mixture minus the sum of the pounds of peanuts and cashews. We now make a chart of the given facts:

	N	x	R¢	=	V¢
Peanuts	x		30		$30x$
Cashews	$2x$		75		$150x$
Almonds	$120 - 3x$		120		$120(120 - 3x)$
Mixture	120		75		$75(120)$

II: We have the equation,

The Sum of the Values	=	the Value of the Sum.	
$30x + 150x + 120(120 - 3x)$	=	$75(120)$	[D
$30x + 150x + 14400 - 360x$	=	9000	[C ^ A
$-180x + 14400$	=	9000	∝ -14400
$-180x$	=	-5400	∝ ÷ (-180)
x	=	30	

Thus, the mixture contains 30 pounds of peanuts, 60 pounds of cashews, and 30 pounds of almonds; which was not one of the three choices offered.

III:

	N	x	$R	=	$V
P	30		.30		9.00
C	60		.75		45.00
A	30		1.20		36.00
	120		.75		90.00

Please go on to the next question.

Question 16

Apply your knowledge and solve the following problem:

A confectioner has two boxes each containing an equal number of lemon drops. From one box, the lemon drops sell three for 1¢ , and from the second box; the lemon drops sell two for 1¢ . The two boxes are combined and the mixture now sells at five for 2¢ . When all the lemon drops are sold, the owner discovers that if he had sold the lemon drops separately, he would have made 5¢ more.

How many lemon drops did each box contain?

Select the letter next to the correct answer.

(A) 300

(B) 200

(C) More than 120 , but less than 180.

(D) Cannot be determined.

We do not agree.

The phrase

$$3abc$$

is a monomial since it consists of the product of a constant and variables.

Please return to page 176/2 and try this question again.

166
1

Yes, your choice is correct.

The phrase

$$5\,\frac{x}{y}$$

is not a monomial. In a monomial, we may not have division by a variable. We may, however, call $5\,\frac{x}{y}$ the quotient of two monomials.

Furthermore, the phrase

$$x + 1$$

consisting of the sum of two terms is not a monomial; it is a binomial.

Please go on to question 2 below.

166
2

Question 2

Recognize which of the following phrases are binomials.

I: $2a^2$ II: $a^2 - b^2$ III: $\frac{x + y}{x - y}$ IV: $xyz + 1$

Select the letter next to the correct answer.

(A) I and III

(B) II and III

(C) II and IV

(D) II , III , and IV

The degree of a monomial is the sum of the degrees in each of the variables. Thus, for example, the degree of

$$8a^3b^2c^4 \text{ is } 3 + 2 + 4 = 9$$

The degree of the coefficient is zero and can be ignored in this calculation.

Please return to page 171/2 and try this question again.

We do not agree. The polynomial

$$5x + 2xy + 4y$$

does not contain any similar terms and, therefore, can't be simplified.

Please return to page 175/2 and try this question again.

168/1

We do not agree. The sum of $\frac{1}{2}a$ and a does not equal $\frac{1}{2}a$.

When a letter is written without a coefficient, the letter is understood to have the coefficient 1. Thus,

$$\frac{1}{2}a + a \quad \text{is the same as} \quad \frac{1}{2}a + 1a = (\frac{1}{2} + 1)a$$

A second error was made

$$-\frac{1}{3}b + b \neq -\frac{2}{3}b$$

You should use the sign of the larger absolute value.

$$(b = \frac{3}{3}b)$$

Please return to page 172/2 and try this question again.

168/2

Did you arrange each of the polynomials in decreasing degree of the variable a ? Let us do so now

$5 - a + a^2$ should be written as $a^2 - a + 5$

$2a + 7 - 3a^2$ should be written as $-3a^2 + 2a + 7$ and

$4a^2 - 7a - 1$ is already in decreasing degree of a.

Please continue.

Please return to page 189/2 and try this question again.

In the word "trinomial" the "tri" means "three" ; the rest of the word can be interpreted to mean "monomial"; that is, a trinomial consists of three monomials. But in the given expression we see $\frac{"z"}{x}$ which we recall is not considered a monomial.

Therefore, this choice must be rejected.

Please return to page 176/2 and reconsider the question.

But at least you were consistant. You made the same error in subtracting monomials.

$4c^2 - c^2$ does not mean drop the c^2 . Rather it means

$$4c^2 - c^2 = 3c^2$$

We do not agree. The expression

$$4c^2 - c^2 + 3c^2$$

consists of three similar terms which can be combined. Thus,

$$4c^2 - c^2 + 3c^2 = c^2 (4 - 1 + 3) = 6c^2$$

Please return to page 185/2 and try this question again.

170
1

Very good. You made the correct choice.

The polynomial

$$x^3 + 3x^2y - 3xy - y^3$$

is written in decreasing degree of the variable x.

The first term is x^3

the second term has x^2

and the third term has x^1

The last term is a constant, and a constant can be considered to be of zero degree. y^3, not containing the variable x, is considered a constant relative to x.

Please go on to question 6 below.

170
2

Question 6

Recognize and select the letter next to the polynomial which is written in increasing degree of the variable x.

(A) $x + 2x^2 + 3x^3 + 4$

(B) $xyz + x^2y + x^3z + \frac{1}{4}x^8$

(C) $3x^2 - 4x + 5$

(D) $a + bx + cx + dx$

Yes, your choice is correct. "Bi" is a prefix meaning two. The phrases

$$(a^2 - b^2) \text{ and } (xyz + 1)$$

each consists of two monomials, and by definition, a binomial is a phrase consisting of two monomials combined by addition or subtraction.

Please go on to question 3 below.

Question 3

Apply the proper principle and select the monomial of highest degree from the following:

I: $\frac{x^8}{4}$ II: $6x^4y$ III: $10xyz^3$ IV: $2x^2y^2z^2$

Select the letter next to the correct answer.

(A) I

(B) II

(C) III

(D) IV

172
1

Very good. You made the correct choice.

To add two polynomials, we use various combinations of the COMMUTATIVE, ASSOCIATIVE, and DISTRIBUTIVE PROPERTIES to combine their similar terms.

Thus,

$$(3x - 2y - 7) + (5 + 3y + 8x)$$

$$3x + 8x + 3y - 2y + 5 - 7 \quad [\text{ COMMUTATIVE}$$

$$(3x + 8x) + (3y - 2y) + (5 - 7) \quad [\text{ ASSOCIATIVE}$$

$$11x + y - 2$$

Hence, the sum is

$$11x + y - 2$$

Please go on to question 10 below.

172
2

Question 10

Apply the proper principles and find the sum of the polynomials,

$$\left(\frac{1}{2}a - \frac{1}{3}b - \frac{1}{4}\right) \text{ and } (a + 1 + b)$$

Select the letter next to the correct answer.

(A) $\frac{3}{2}a + \frac{2}{3}b + \frac{3}{4}$

(B) $\frac{3}{2}a + \frac{2}{3}b - \frac{3}{4}$

(C) $\frac{1}{2}a - \frac{2}{3}b + \frac{3}{4}$

(D) None of these.

We do not agree. One of the letters does have the correct answer next to it.

Please return to page $\frac{185}{2}$ and try this question again.

We do not agree.

You seem to have lost the term

$$-x^3$$

This term cannot disappear unless its opposite

$$+x^3$$

appears in one of the polynomials.

Please return to page $\frac{179}{2}$ and try this question again.

174
1

Did you check your answer? Most students just "go over the work" mentally verifying each step until they are satisfied that their answer is correct. The main defect of this system is that they are quite likely to make the same mistake again, especially if they have a misconception about the proper rule.

A good method that will catch errors of that sort is a little more time consuming, but objective (in that a mistake can be seen). It is described now:

(1) Choose convenient small values for the variables.

(2) Substitute them in the original form of the problem. Find the value of the original expression.

(3) Now substitute the same values for the variables in the answer that is to be checked.
Since the answer is supposed to be an equivalent form (though simplified) of the original expression, it should have the same evaluation.

(4) If the answer does not check with the original, either

(a) there was a mistake in arithmetic in the checking process, or
(b) there is a mistake in the algebraic work.

In either case, it is a danger signal that something is wrong.

Please return to page 183/2 and try this problem again, checking your work.

174
2

Note that a minus before a symbol of inclusion changes each term within the symbol to its opposite. Thus,

$$-(5x - 4) = -5x + 4$$

This follows the principle that

$$a - b \text{ means } a + (-b)$$

That is subtraction accomplished by adding the additive inverse.

Please return to page 192/2 and try this question again.

Very good. You made the correct choice. The polynomial

$$xyz + x^2y + x^3y + \frac{1}{4}x^8$$

is written in increasing degree of the variable x .

In the first term, x is of the first degree.

In the second term, x is of the second degree.

In the third term, x is of the third degree.

In the fourth term, x is of the eighth degree.

Please go on to question 7 below.

Question 7

Recognize which of the following polynomials can be simplified.

I: $5ab + 5a - b + 5$

II: $8 + k - 3$

III: $7y^3 - ay - by^2 + 17y^3$

IV: $5x + 2xy + 4y$

Select the letter next to the correct answer.

(A) I and II

(B) III and IV

(C) II and IV

(D) II and III

176
1

Very good. You made the correct choice. The degree of the monomial

$$\frac{x^8}{4}$$

is higher than the degree of the other monomials. Thus,

the degree of $\frac{x^8}{4}$ is 8 (the 4 can be considered as the coefficient $\frac{1}{4}$ and has a zero degree)

the degree of $2x^2y^2z^2$ is $2 + 2 + 2 = 6$

the degree of $6x^4y$ is $4 + 1 = 5$

the degree of $10xyz^3$ is $1 + 1 + 3 = 5$

Please go on to question 4 below.

176
2

Question 4

Recognize which of the following phrases are trinomials.

I: $a^3 + b^3 + c^3$

II: $3abc$

III: $a^2 - b^2 + 2$

IV: $\frac{x}{y} + \frac{y}{3} - \frac{z}{x}$

Select the letter next to the correct answer.

(A) I and II

(B) I and III

(C) II and III

(D) III and IV

Very good. You made the correct choice. Expressions are simplified by combining similar terms. Instead of writing

$$(11 + 7 - 9)$$

it is certainly preferable to write 9 . We have,

I:	$4c^2 - c^2 + 3c^2 = 6c^2$	all the terms are similar
II:	$18k - 13k + 4m = 5k + 4m$	the first two terms are similar
III:	$a + 3a - 5a = -a$	all the terms are similar
IV:	$18mr - mr + 3m = 17mr + 3m$	the first two terms are similar

Please go on to question 9 below.

Question 9

Apply the principles of addition of polynomials and find the sum of:

$$(3x - 2y - 7) \text{ and } (5 + 3y + 8x)$$

Select the letter next to the correct answer.

(A) $11x + y - 2$

(B) $8x - y - x$

(C) $11x - y + 2$

(D) $11x - 5y - 12$

178
1

Re-arrange the two polynomials with the similar terms one under the other. Then very carefully apply the rules for addition of signed numbers.

We do not agree. One of the letters does have the correct answer next to it.

Please return to page 172/2 and try this question again.

178
2

It is important that you know the rule for subtracting two polynomials. The rule states: to subtract two polynomials, add the opposite of each term of the subtrahend to the minuend. In case the words subtrahend and minuend are not to your liking, let us state the rule without using these words. Each term of the polynomial that is to be taken from another, is changed to its opposite, and then the polynomials are added.

Thus, for example, if the polynomial

$$2x - 3y$$

is to be subtracted from

$$7x + 8y$$

we change each term of $2x - 3y$ to its opposite and then add the polynomials.

$$\begin{array}{r} 7x + 8y \\ 2x - 3y \end{array} \qquad \begin{array}{r} 7x + 8y \\ -2x + 3y \\ \hline 5x + 11y \end{array}$$

Please return to page 197/2 and try this question again.

In order to transform the left side of the given inequality into the left side of this choice, we subtract 6 from both sides and then multiply both sides by -2 . Then we have

$$3x + 6 > -9 \quad \not{-6}$$

$$3x > -15 \quad \not{\cdot(-2)}$$

$$-6x < +30$$

Note that the order of the inequality reversed since we used a negative multiplier.

The right side was transformed, therefore, and the derived inequality is identical with your choice.

Please proceed to question 13 below.

Question 13

Apply the proper principle and find the sum of the polynomials,

$$9 - x^3 + 5x - x^2 \quad \text{and} \quad 2x^2 + x^4 - 7 - 3x$$

Select the letter next to the correct answer.

(A) $x^4 - x^3 + x^2 + 2x + 2$

(B) $x^4 + x^2 + 2x + 2$

(C) $2x^4 + x^3 + x^2 + 2x + 2$

(D) None of these.

180
1

You are only partly right.

To be simplified, polynomials must have similar terms.
The polynomial

$$8 + k - 3$$

has the similar terms 8 and -3, but the polynomial

$$5x + 2xy + 4y$$

does not have any similar terms. In order for terms to be similar, they must have the same variables raised to the same powers.

Please return to page 175/2 and try this question again.

180
2

We do not agree. To find the difference of the profits of the two stores, we need to find the following difference:

$$(15s^2 - 74s + 40) - (9s^2 + 12s - 22)$$

Change each term of the second polynomial to its opposite and add.

Please return to page 195/2 and try this question again.

Very good. Your choice is correct.

A trinomial is a phrase consisting of three monomials. Thus,

$$a^3 + b^3 + c^3$$

is a trinomial, and

$$a^2 - b^2 + 2$$

is also a trinomial.

Note: A constant is considered a term of degree zero. Therefore:

$x + 2$ is a binomial

$x + y + 2$ is a trinomial

Please go on to question 5 below.

Question 5

Recognize and select the letter next to the polynomial which is written in decreasing degree of the variable x .

(A) $x^3 - y^3 + 3xy^2 - 3x^2y$

(B) $x^3 + 3x^2y - 3xy^2 - y^3$

(C) $3x^2y - 3xy^2 + x^3 - y^3$

(D) $-y^3 + 3xy^2 - 3x^2y + x^3$

182
1

We do not agree. The question can be written as follows:

$$\begin{array}{r} 5x - 6y + z \\ \underline{3x + y - z} \end{array}$$

where the lower polynomial is to be subtracted from the one above. The rule for subtracting one polynomial from another is to add to the minuend the opposite of each term of the subtrahend.

In other words, we change each of the signs of the lower polynomial and then add. The question then reduces to finding the sum of

$$(5x - 6y + z) \quad \text{and} \quad (-3x - y + z)$$

Please return to page 190/2 and try this question again.

182
2

Did you check your answer?

If $y = -2$

is a solution of the given equation, then

$$2(-2) - [5(-2) - (6 \cdot [-2] + 2)]$$

should equal: $10 - (-2)$ or 12

$$\begin{array}{l} -4 - [-10 - (-12 + 2)] \\ -4 - [-10 - (-10)] \\ -4 - [-10 + 10] \\ -4 - 0 \\ -4 \\ -4 \neq 12 \end{array}$$

Please return to page 196/2 and try this question again.

Very good. You made the correct choice. To add two polynomials, we combine their similar terms.

$\frac{1}{2}a + a = (\frac{1}{2} + 1)a = (\frac{3}{2})a$

$-\frac{1}{3}b + b = (-\frac{1}{3} + 1)b = \frac{2}{3}b$ and

$-\frac{1}{4} + 1 = \frac{3}{4}$

$\frac{3}{2}a + \frac{2}{3}b + \frac{3}{4}$

$\frac{1}{2}a - \frac{1}{3}b - \frac{1}{4}$

$a + b + 1$

Hence, the sum is $\frac{3}{2}a + \frac{2}{3}b + \frac{3}{4}$

One technique is to write the polynomials so that the similar terms are directly under each other. In that way, the problem becomes a series of addition of monomials.

Please go on to the next question below.

Question 11

Apply your knowledge and find the sum of the polynomials,

$(5x^2 + x - 5)$, $(x^2 - 4x + 7)$, $(3x - 7x^2 - 8)$

Select the letter next to the correct answer.

(A) $-x^2 - 6$

(B) $-x^2 + x - 6$

(C) $-2x^2 - 6$

(D) $-x^2 - x - 6$

3 OF 3

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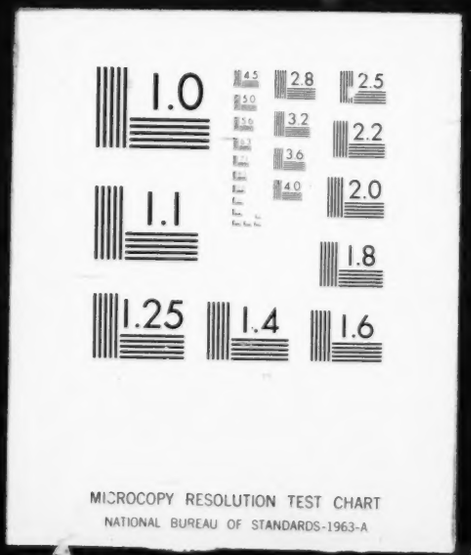

184/1

We do not agree. Each term of the subtrahend should be changed to its opposite. Thus,

$$a - b - c \quad \text{should be changed to} \quad -a + b + c$$

After this is done, the polynomials are <u>added</u>.

Please return to page 197/2 and try this question again.

184/2

Do you understand the question? First, you are asked to find the sum of the polynomials

$$2x^2 + 5x - 3 \quad \text{and} \quad 3x^2 - 8x + 2$$

After you obtain this sum, you are to subtract the polynomial

$$x^2 - 3x + 4 \quad \text{from this sum.}$$

Remember, a quick check can be accomplished by letting

$$x = 3$$

Please return to page 188/2 and try this question again.

Very good. You made the correct choice.

The polynomial $8 + k - 3$

has 8 and - 3 as similar terms; it becomes

$$k + 5$$

The polynomial $7y^3 - ay - by^2 + 17y^3$

has the similar terms $7y^3$ and $17y^3$

Terms that have the same variables raised to the same powers are called similar

The polynomial becomes $24y^3 - by^2 - ay$

after being simplified.

Please go on to question 8 below.

Question 8

Apply your knowledge and write each of the following phrases in simplest form. Select the letter next to the correct answer.

I. $4c^2 - c^2 + 3c^2$
II $18k - 13k + 4m$
III $a + 3a - ra$
IV. $18mr - mr + 3m$

(A) I. $6c^2$
II. 9km
III. -a
IV. 20mr

(B) I. $6c^2$
II. $5k + 4m$
III. -a
IV. $17mr + 3m$

(C) I. $4 + 3c^2$
II. $5 + 4m$
III. $4a^2 - 5a$
IV. $18 + 3m$

(D) None of these

186/1

You must have made a careless error.

Note that x^4 appears only in the second polynomial.

Thus the answer cannot begin with $2x^4$.

Please return to page 179/2 and try this question again.

186/2

We do not agree. Let us do a problem with a similar situation. Find the solution set of the equation,

$$(4y - 5) - (2y - 8) = 11$$

Since there is no minus sign before

$$(4y - 5)$$

the parentheses are removed without any change. Since there is a minus sign before

$$(2y - 8)$$

each term inside the parentheses is changed to its opposite. Thus, we have

$$4y - 5 - 2y + 8 = 11$$

Combining similar terms, we get

$$\begin{aligned} 2y + 3 &= 11 \\ 2y &= 8 \\ y &= 4 \end{aligned}$$

Please return to page 192/2 and try this question again.

We do not agree. Recall your work with symbols of inclusion; the procedure is to remove the innermost symbol first. Thus,

$$2y - [\ 5y - (\ 6y + 2\ )\ ] =$$

$$2y - [\ 5y - 6y - 2\ ] =$$

$$2y - [\ -y - 2\ ]$$

Please continue.

Please return to page 196/2 and try this question again.

You understand that these two polynomials must be subtracted. But the entire second polynomial must be subtracted from the first. If you were wondering how to handle the months and the days, perhaps a reminder to convert one year into twelve months, or one month into thirty days, and to adjust the form of the other boy's age as is needed, would be helpful.

Please return to page 191/2 and try this procedure again.

188
1

Very good. You made the correct choice. To find the difference of the profits of the two stores we have to subtract

$9s^2 + 12s - 22$ from $15s^2 - 74s + 40$

We change each term of the first of these polynomials to its opposite, and perform the following addition:

$$\begin{array}{r} 15s^2 - 74s + 40 \\ 9s^2 + 12s - 22 \\ \hline \end{array} \qquad \begin{array}{r} 15s^2 - 74s + 40 \\ -9s^2 - 12s + 22 \\ \hline 6s^2 - 86s + 62 \end{array}$$

Please go on to question 17 below.

188
2

Question 17

Apply the proper principle and subtract

$x^2 - 3x + 4$

from the sum of $2x^2 + 5x - 3$

and $3x^2 - 8x + 2$

Select the letter next to the correct answer.

(A) $4x^2 - 3$

(B) $4x^2 - 6x - 5$

(C) $4x^2 - 5$

(D) $4x^2 + 3$

Very good. You made the correct choice. We shall demonstrate the two methods generally used:

Method I: (Using the COMMUTATIVE and ASSOCIATIVE PROPERTIES OF ADDITION)

$$(5x^2 + x - 5) + (x^2 - 4x + 7) + (3x - 7x^2 - 8)$$

$$(5x^2 + x^2 - 7x^2) + (x - 4x + 3x) + (-5 + 7 - 8)$$

$$-x^2 + 0x - 6$$

$$-x^2 - 6$$

Method II: (Rearranging the polynomials one under the other with similar terms in columns.)

$$\begin{array}{rrrr} 5x^2 & + x & - 5 & \\ x^2 & - 4x & + 7 & \\ -7x^2 & + 3x & - 8 & \\ \hline -x^2 & + 0x & - 6 & \end{array}$$

Check: Let $x = 3$

$$\begin{array}{rrrr} 5(9) & + (3) & - 5 & = 43 \\ (9) & - 4(3) & + 7 & = 4 \\ -7(9) & + 3(3) & - 8 & = -62 \\ \hline -(9) & & - 6 & = -15 \end{array}$$

Thus, the correct answer is:

$$-x^2 - 6$$

Please go on to question 12 below.

Question 12

Apply your knowledge and find the sum of the polynomials,

$5 - a + a^2$ $\quad 2a + 7 - 3a^2$ and $4a^2 - 7a - 1$

Select the letter next to the correct answer.

(A) $11a^2 - 4a + 2$

(B) $-4a^2 + 2a + 11$

(C) $2a^2 + 11a - 4$

(D) $2a^2 - 6a + 11$

190
1

Very good. You made the correct choice.

To subtract one polynomial from another, add the minuend to the opposite of each term of the subtrahend.

Thus,

Method I:

$$3a + b - c - (a - b - c)$$

$$3a + b - c + (-a + b + c) \quad [\ C \wedge A$$

$$(3a - a) + (b + b) + (-c + c)$$

$$2a + 2b$$

Method II:

$$\begin{array}{r} 3a + b - c \\ -(a - b - c) \\ \hline \end{array} \qquad \begin{array}{r} 3a + b - c \\ -a + b + c \\ \hline 2a + 2b \end{array}$$

Method III: A convenient device is to circle the original sign of each term in the subtrahend and write the appropriate sign above it.

$$\begin{array}{r} 3a + b - c \\ \overset{-}{\oplus}a \ \overset{+}{\ominus}b \ \overset{+}{\ominus}c \\ \hline 2a + 2b \end{array}$$

This becomes a "check" that each sign was changed.

Please go on to question 15 below.

190
2

Question 15

Apply your knowledge and perform the following subtraction:

$$(5x - 6y + z) - (3x + y - z)$$

Select the letter next to the correct answer.

(A) $2x - 5y$

(B) $2x - 5y + 2z$

(C) $8x - 5y$

(D) $2x - 7y + 2z$

Very good, You made the correct choice.

We start by simplifying the inner parentheses first.

$$2y - [\,5y - (\,6y + 2\,)\,] = 10 - y \quad [-(a + b) = -a - b$$

$$2y - [\,5y - 6y - 2\,] = 10 - y \quad [\text{ Combine}$$

$$2y - [\,-y - 2\,] = 10 - y \quad [-(-a - b) = a + b$$

$$2y + y + 2 = 10 - y \quad [\text{ Combine}$$

$$3y + 2 = 10 - y \quad \langle + y$$

$$4y + 2 = 10 \quad \langle - 2$$

$$4y = 8 \quad \langle \div 4$$

$$y = 2$$

Please go on to question 20 below.

Question 20

Relate to the proper principle and answer the following question:

"A computer is programmed to perform additions and subtractions on ages if they are expressed as polynomials. If one boy's age is expressed as,

$$11y + 3m + 2d$$

and a second boy's age is recorded as,

$$6y + 7m + 3d$$

choose the letter next to the answer the computer would produce if it were asked to find the difference in their ages."

(Let 1 month = 30 days) Note: y = year
m = month
d = day

(A) $5y + 4m + 1d$

(B) $4y + 10m + 5d$

(C) $4y + 7m + 29d$

(D) $5y + (-4m) + (-1d)$

192
1

Very good. You made the correct choice. We start by finding the sum of

$$(2x^2 + 5x - 3)$$

and $$\underline{(3x^2 - 8x + 2)}$$

$$5x^2 - 3x - 1$$

From this sum, we subtract

$$(x^2 - 3x + 4)$$

Thus, we have

$$(5x^2 - x^2) + (-3x + 3x) + (-1 - 4) = 4x^2 - 5$$

Please go on to question 18 below.

192
2

Question 18

Apply your knowledge and find the solution set of the equation

$$(8x + 3) - (5x - 4) + (x - 2) = 17$$

Select the letter next to the correct answer.

(A) $x = 2$

(B) $x = 2\frac{1}{2}$

(C) $x = 4$

(D) $x = 3$

$\frac{193}{1}$

We do not agree.

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Check by letting

$$x = 3$$

Please return to page $\frac{179}{2}$ and try this question again.

$\frac{193}{2}$

Your choice is incorrect. Would you like a head start? Here's how you start:

$$2y - [\,5y - (\,6y + 2\,)\,] = 10 - y$$

$$2y - [\,5y - 6y - 2\,] = 10 - y$$

$$2y - [\,-\,y - 2\,] = 10 - y$$

Please continue.

Please return to page $\frac{196}{2}$ and complete the problem.

VI

194
1

It is always a good idea to check your answer. Let us do so.

If $x = 4$ is the solution of the given equation, then

$$(8x + 3) - 5(x - 4) + (x - 2) \stackrel{?}{=} 17$$

$$8(4) + 3 - 5(4 - 4) + (4 - 2) \stackrel{?}{=} 17$$

$$32 + 3 - 5(0) + 2 \stackrel{?}{=} 17$$

$$37 \neq 17$$

Hence, $x = 4$ is not a solution.

Please return to page 192/2 and try this question again.

194
2

Well you did subtract the two polynomials. But are you satisfied with the form of the answer? An age is not easily understood if it involves positive years and a negative number of months and days. Don't you think that you could handle the adjustment in the same way that the computer programmer would have to in order to eliminate the negatives? This calls for a substitution of twelve months for one year, and thirty days for one month.

Suppose you do this and then choose your answer.

Please return to page 191/2 and try this problem again.

Very good. You made the correct choice.

We have to subtract

$$3x + y - z \text{ from } 5x - 6y + z$$

To do this, we change each term of

$$3x + y - x$$

to its opposite and then add the polynomials. Thus, the equation reduces to finding the sum of

$$(5x - 6y + z) \text{ and } (-3x - y + z)$$

which equals $(5x - 3x) + (-6y - y) + (z + z) =$

$$2x - 7y + 2z$$

Alternate method:

$$\begin{array}{r} 5x - 6y + z \\ \bar{\oplus}\ 3x\ \bar{\oplus}\ y\ \overset{+}{\ominus}\ z \\ \hline 2x - 7y + 2z \end{array}$$

Please go on to question 16 below.

Question 16

Apply your knowledge and answer the following question:

In one department store, a research project found that the profit on sales can be computed by the expression

$$15s^2 - 74s + 40$$

In a branch store the profit on sales is computed by the expression

$$9s^2 + 12s - 22$$

Which expression shows the difference of the profits between the two stores, if the branch made less profit?

(A) $24s^2 - 86s + 62$

(B) $6s^2 - 62s + 28$

(C) $6s^2 - 86s + 62$

(D) None of these.

196
1

Very good. You made the correct choice.

We have to solve the equation

$$(8x + 3) - (5x - 4) + (x - 2) = 17$$

Note that a minus sign before a symbol of inclusion changes each term within the symbol to its opposite. Thus,

$$8x + 3 - 5x + 4 + x - 2 = 17 \quad [\ C\hat{}A$$

Combining similar terms, we obtain

$$4x + 5 = 17 \quad \propto -5$$

$$4x = 12 \quad \propto \div 4$$

$$x = 3$$

Please go on to question 19 below.

196
2

Question 19

Apply your knowledge and find the solution set of the equation,

$$2y - [\ 5y - (6y + 2)\ ] = 10 - y$$

Select the letter next to the correct answer.

(A) $y = -9$

(B) $y = -2$

(C) $y = 3$

(D) $y = 2$

Very good. You made the correct choice. We start by rewriting the polynomials in decreasing degree of the variable x. Thus,

$$9 - x^3 + 5x - x^2$$

is written as $-x^3 - x^2 + 5x + 9$

and $2x^2 + x^4 - 7 - 3x$

is written as $x^4 + 2x^2 - 3x - 7$

Note that the first polynomial does not have a term of degree 4 and the second polynomial does not have a term of degree 3. We, therefore, write the polynomials as follows:

		Check: let $x = 3$
$x^4 + 2x^2 - 3x - 7$	81	$+ 18 - 9 - 7 = 83$
$-x^3 - x^2 + 5x + 9$		$-27 - 9 + 15 + 9 = -12$
$x^4 - x^3 + x^2 + 2x + 2$	81	$-27 + 9 + 6 + 2 = 71$

Notice that a space was left open between x^4 and $2x^2$ to make room for the term $-x^3$ in the second row. Adding similar terms, we get the sum

$$x^4 - x^3 + x^2 + 2x + 2$$

Note: We let $x = 3$ because it's the smallest number that will not disguise errors. $x = 0$ would eliminate all variables; $x = 1$ would ignore exponent errors, and $x = 2$ makes errors such as

$$2 \times 2 = 2 + 2$$

Please go on to question 14 below.

Question 14

Apply your knowledge and subtract

$$(a - b - c)$$

from

$$3a + b - c$$

Select the letter next to the correct answer.

(A) $2a - 2b$

(B) $2a + 2b - 2c$

(C) $2a + 2b$

(D) $4a - 2c$

Very good. You made the correct choice. Let's review a method that could have been used in solving this problem:

Age of 1st boy: $11y + 3m + 2d$

Age of 2nd boy: $\ominus\ 6y\ \ominus\ 7m\ \ominus\ 3d$ We change signs and add.

The difference is: $5y + (-4m) + (-1d)$

Since this form of the answer is not acceptable because of the negative values, we must make an adjustment in the form by substituting equivalent values. We note that

$1y = 12m$
and that $1m = 30d$
also $5y = 4y + 1y$

Therefore, we can write:

$5y + (-4m) + (-1d)$ [Sub: $5y = 4y + 1y$

$4y + 1y + (-4m) + (-1d)$ [Sub: $1y = 12m$

$4y + 12m + (-4m) + (-1d)$ [ASSOCIATE PROPERTY

$4y + (12m - 4m) + (-1d)$ [Collect

$+8m + (-1d)$ [Sub: $8m = 7m + 1m$

$4y + (7m + 1m) + (-1d)$ [Sub: $1m = 30d$

$4y + 7m + 30d + (-1d)$ [ASSOCIATIVE PROPERTY

$4y + 7m + (30d - 1d)$ [Collect

$4y + 7m + 29d$

Note: This answer is in acceptable form.

You have now finished this Segment. Hand in your PUNCH CARD.
You should have entered in your NOTEBOOK the following definitions and formulas:

(1) To add two polynomials, combine their similar terms.

(2) To subtract one polynomial from another, add to the minuend the opposite of each term of the subtrahend.

You should now be able to complete the balance of your HOMEWORK ASSIGNMENT. Review all notes in preparation for the Volume Test.

RE'ISED - Decem[illegible], 1969

PROGRAMMED MATHEMATICS CONTINUUM
ALGEBRA - LEVEL ONE

ERRATA SHEET
VOLUME 6

Attach to
Back Cover

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected.

Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE	MISPRINT	CORRECTION	CHECK WHEN CORRECTION MADE
$\frac{19}{1}$	$\frac{11}{2}$	$\frac{19}{2}$	
129	(Page direction Omitted)	Please return to Page $\frac{139}{1}$	
$\frac{69}{1}$	(B)I 3 (C)I 3 (D)I 3	(B)I 13 (C)I 13 (D)I 13	
$\frac{78}{2}$	find angles of triangle (C) 21 (D) 28	find smallest angle of triangle (C) 28 (D) 21	